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Treball Final de grau

Formalizing the harmony of jazz standards: recursive structural modes and generative grammars

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Abstract

Català

L'harmonia dels estàndards de jazz fonamenta molts dels aspectes de la interpretació, però la varietat entre estils dificulta trobar una lògica que descrigui de manera general la dinàmica entre acords. Aquest treball se centra en dos objectius: contextualitzar dos models harmònics i el seu àmbit d'aplicació, i proposar-ne modificacions a partir de la interacció dels dos. Per això, introduirem un seguit de propietats teòriques suplementàries, inferirem algunes característiques a partir de l'anàlisi d'exemples i explorarem construccions relacionades que ressaltin funcionalitats determinades dels sistemes. Seguirem els articles dels autors dels models (De Jong, Noll; Rohrmeier), així com el manual de referència en computació de Lewis i Papadimitriou.

English

The harmony of jazz standards lays the foundations of many aspects of the interpretation. However, the variety within the corpus makes it complicates the attempts to find a logic behind the general dynamics between chords. This work focuses on two main objectives: to contextualize two harmonic models and their scope, and propose modifications based on the interaction between them. For that purpose, we introduce a collection of supplementary theoretical properties, we infer some features from the analysis of examples and we explore related constructions which emphasize certain functionalities of the systems. We follow the authors' papers on their models (De Jong, Noll; Rohrmeier), as well as Lewis' and Papadimitriou's reference textbook on computation theory.

Español

La armonía de los estándares de jazz fundamenta muchos aspectos de la interpretación, pero la variedad entre estilos dificulta poder encontrar una lógica que describa de manera general la dinámica entre acordes. Este trabajo se centra en dos objectvos: contextualizar dos modelos armónicos y su ámbito de aplicación, y proponer modificaciones a partir de la interacción de los dos. Para tales fines introduciremos un listado de propiedades teóricas suplementarias, inferiremos algunas características a partir del análisis de ejemplos y exploraremos construcciones relacionadas que resalten funcionalidades determinadas de los sistemas. Seguiremos los artículos de los autores de los modelos (De Jong, Noll; Rohrmeier), así como el manual de referencia en computación de Lewis y Papadimitriou.

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Introduction

In many situations concerning musical practice or analysis, a certain celerity to process harmonic material is essential. Such an understanding usually stems from the ability to recognize and interpret structures which underlie chord sequences. Despite the fact that a tradition of jazz standards has existed for decades, there is still no complete method to recreate their harmony consistently.

In this work, we explore two approaches to tonal harmony which can be restricted to that of some jazz tunes: structural modes and a generative grammar of chord progressions. Our aims are of both theoretical and practical nature. On the one hand, we set out to describe the assumptions which each model imposes to their conception of tonal harmony, as well as the effects which these have on the body of studied works. On the other hand, we extend the models by means of some minor modifications, in order to reach more holistic interpretations and ultimately propose a mixed system which conveys information from the rest.

We mainly employ two key resources to address these challenges: the parsing of the harmony behind some jazz standards according to our models, and the manipulation of grammar and automata as mathematical constructions which can set our assumptions and views on tonality closer to the actual musical phenomenon.

The followed musical bibliography consists on a series of papers from the respective authors of the systems, compiled throughout the years. Regarding the computational and mathematical topics, Lewis' and Papadimitriou's book on computation is the main reference, along with miscellaneous lecture notes and articles.

I would like to express my deepest gratitude to my family and friends for their unrelenting support. And, without a doubt, I want to acknowledge my appreciation for the commitment of the musicians I worked with, learned from and played along with, especially when it has come to my involvement in musical theory.

Chapter 1

Theoretical framework

Harmony has been one of the predominant focuses for the analysis and teaching of mainstream occidental classical music and, to some extent, its predecessors and the genres it influenced. Accordingly, many interpretations and classifications have been proposed in regard to pitch class material: in a fair amount of currents, it is common practice to identify chords as individual entities (either in a specific instant or in measure spans) which then play a certain role within a structure. Whether this relationship occurs in terms of functional harmony in a tonality or a mode, or as non-functional aggregates along a succession of pitch class regions, the formal aspect has been studied and developed broadly by different schools of thought.

Nevertheless, there is still much to be explored as to how functionally harmonic material –and, in particular, chords– interacts in a local scale: on the one hand, one can try to argue how the progressions and movements can arise between several elements within a mode by suggesting a model beyond a simple taxonomy, which could fail to be expanded to higher structural levels by means of its own resources. On the other hand, there is the possibility to define a logic which could inherently substantiate both some of the chord successions and the harmonic behavior of the entire subject of study.

This chapter presents two recently developed systems ([Roh20], [DN18]) which tackle these challenges, thus laying the foundations for a more exhaustive discussion of both their features and limitations, and the upgrades that can be implemented to further leverage their distinctive perspective on functional harmony. Additionally, some theoretical notions about automata are introduced as a tool to refine some of the models of analysis.

1.1 Context

When it comes to determining the form of a piece or a tune, few aspects apart from thematic analysis are as effective as being able to delimit the modes and tonalities which make up the entirety of the timeline. For that purpose, pinpointing harmonic resources such as turnarounds or cadences is usually a pragmatic starting point. Conversely, the tonal region which some melody or material may take place in often outlines the specificities of their underlying chords and scales. One of the ongoing directions in current musical theory is trying to make sense out of certain occurrences and establish a system which could tie the harmonic subtleties to a unified reference frame of the musical work in question.

One of the models which we will be delving into finds its origins no later than the medieval era. Embedded structural modes are a special case of Carey and Clampitt's well-formed scales ([CC89]), and their usage as bare pillar notes can be traced back to the times of Guido de Arezzo and hexachordal scales. In opposition to a transformative theory of chords as a collection of pitches such as Riemann's, the authors of the model [DN18] focus on the movement of the fundamental bass while reformulating the assignation of tonal functions to the different positions of the scale.[1] They do so by continuing some of the ideas from Handschin and Meeus regarding progressions and direction in tonality.

The other system which we will go over, proposed by Rohrmeier in [Roh20], follows the Chomskyan tradition of generative grammars and aims to provide a framework to encompass various levels of complexity within the same studied work. This

[1] Even though some other perspective has also been explored in [DNY15] and [HR11].

is an opposing viewpoint to Bayesian and Markovian-oriented grammars, which are still used in musical analysis for a result more centered around statistics and data treatment. Much as Lerdahl and Jackendoff's celebrated Generative Theory of Tonal Music [JL85], the syntax of this model offers an outlook on pieces through hierarchies and tree representations, but it improves previous attempts ([NR15]) by taking into account notions from Neo-Riemannian theory and the assumption of a right-headed harmony. Related developments include [HHT06]'s syntax of tonal cadences or even [Ste99]'s formalization of the 12-bar blues, and one is likely to find links between this branch of analysis and structural modes in more recent papers like [HR11].

Each of the two perspectives will be more susceptible to illustrate the features of a particular kind of harmonical resources, so to begin with it is indispensable to narrow down the kind of music which we will be analyzing. The following sections will introduce the fundamentals of both systems of analysis in a way that we will be able to directly address the description of how they operate, what contexts they can be applied in, what are the assumptions or features that impose to the resulting analysis, and what limitations they may have.

After that, we will be able to analyze some actual music in depth by applying some changes to the models and even suggesting some rudimentary system that involves both of them, which will require to formalize grammars in terms of automata for a more straightforward implementation. Our goal is to achieve or come close to a comprehensive, multilayered type of analysis (such as Schenker's) while maintaining the schemata-like precision which the rules of Rohrmeier's syntax and the bass dynamics in [DN19] feature.

1.2 Embedded structural modes

Structural modes arise as one of the simplest configurations that a well-formed mode can adopt. This allows them to become a representative portion of the entire diatonic mode, while remaining concise enough to replicate their disposition over other degrees, thus creating coverings of the whole chromatic collection where the same note can manifest different tonal functions. One can then define a set of possible bass transformations and derive from these dynamics a system of analysis.

Let us first introduce some notation from [CC89]: consider a chromatic universe identifying pitches modulo the octave, which in our case will be the 12-tone equal temperament. We can contemplate an interval of a fixed size in semitones, and construct a collection of notes by considering a succession of a finite amount of adjacent instances of said interval. After rearranging the resulting pitches within an octave, we obtain a scale *generated by* the interval (which we call the *generator*). Now every interval between two scale degrees can be measured by counting the number of notes in the pertinent segment of the scale (*generic size* or *span*) or through its amount of semitones (*specific size*).

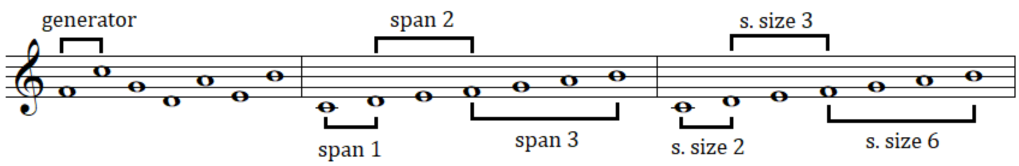

Figure 1: The C major scale is generated by the perfect fifth, an interval of specific size 7. The following measures exemplify, respectively, different generic and specific sizes of intervals.

Well-formed scales are characterized in a variety of ways, but in terms of our previous definitions, the following statements are equivalent for any scale S generated by an interval I (of a fixed specific size):

a) S is well-formed.

b) The generic size of I is identical in each of its instances within S.

c) *Myhill's property*: the sets of specific sizes for the instances of each generic interval in S –besides from the unison and octaves– have cardinality exactly 2.

In order to construct a *structural mode* like [CN11], we only employ two iterations of the same "perfect fifth" interval, which results in a 3-note scale (4 if we count the octave) with two possibilities for the interval sizes in each of the presented metrics: generic sizes 1 and 2 (since 3 represents the octave), and specific sizes 2 and 5. For example, C−D−G−C′. It is easy to check either of the aforementioned conditions to assert that such mode is well-formed. On the one hand, each of the instances of the generating interval is made up of two smaller intervals of respective sizes 2 and 5. On the other hand, to confirm Myhill's property, seconds[2] can either be of specific size 2 or 5, while thirds are a combination of 2+5 or 5+5 semitones.

Having checked that structural scales are well-formed, we can introduce some variations (properly, *modes*) depending on which note we consider to be the first degree: given the structural scale with C−G−D as notes, the *first* mode starts at G (G−C′−D′−G′), the *second* at C (C−D−G−C), and the *third* one at D (D−G−C−D). We may also arrange them within a same octave range, illustrating the modes in a similar way as the tonoi of classic modes.

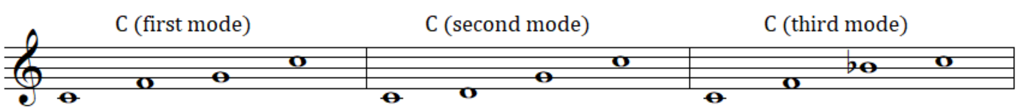

Figure 2: Modes of the structural scales disposed over C as the base note.

Like many well-formed, generated modes, structural scales share some symmetry properties with the diatonic scale. To represent each of the notes of a mode within a single octave, we may proceed by considering either the native generic intervals of second and their two possible specific sizes (that is, a proper ascending scale), or the *fifth/fourth folding*: a combination of the generating interval and its –downward– inversion modulo octave, i.e., ascending 7 semitones and descending 5 semitones, so that both preserve the "fifth's" sharp-ward direction. Let us focus on the first mode as an example (presented in [DN18]), and divide both the scale

[2]As generics intervals in the current mode, rather than in the classical diatonic scale.

and the folding at the diatonic perfect fifth from the base note. Then, the resulting intervals in the succession of notes can be expressed as the concatenation of a single interval from each type used in the other succession of notes. That is, F−D in the fifth/forth folding can be represented by a structural minor second (2 semitones) and a descending structural major second (5 semitones), and C−C′ can be expressed through the generating interval (a diatonic perfect fifth) and its ascending inversion modulo the octave (a diatonic perfect forth).

Figure 3: [DN18] show the duality between a scale arrangement of the first structural mode in C and their fifth/fourth folding.

In the diatonic case, the interval F−F♯ corresponds to an *augmented prime* and is associated with a sharp-ward modulation, which acts as the *dual interval* of the octave. This justifies the character of the structural interval F−D as a *structural augmented prime* and has proven essential to describe the behavior of the structural bass.

In relation to the analysis, we will use the generating interval of the structural mode and its augmented prime to describe the navigation of the fundamental bass (which may differ from the real one). Therefore, we will represent the fundamental bass line through a system of two axes, taking the vertical one for the *progressions* (transformations of the size of the generating interval) and the horizontal one for the *alterations* (augmented prime). We may also use generally non-commutative combinations[3] if the chords are not close enough.

However, one of the most noteworthy virtues of this model of analysis is the interpretation of tonal functions: we assign the *tonic* function (T) to the base note of the mode; the second degree corresponds to the *subdominant* (D), and the third

[3]Depticted by double arrows in the case of two identical transformations.

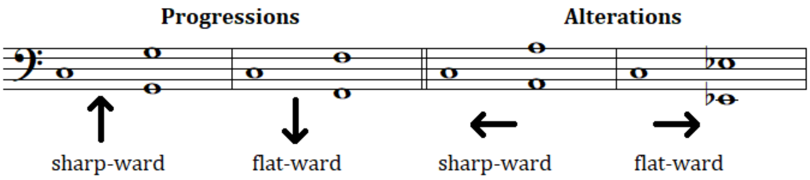

Figure 4: Notation for the movement of the fundamental bass in structural modes.

one to the *dominant* (D). This is compatible with the usual function attribution in diatonic modes, since degrees II and IV work as predecessors of the dominant par excellence, and $\flat VII$ is often used as a backdoor dominant. Having set out the tools for the analysis, the interest lies in the interpretation of the tonal areas and the selection of the appropriate modes. For example, we may consider a second mode (which includes degree notes II, V and I) but ultimately need to notate the existence of a degree IV, in which case we will use $\flat S$ as a flat-ward alteration of the present subdominant, even if IV was reached from the tonic. This also reflects the fact that three flat-ward progressions $\downarrow$ can equate a flat-ward alteration $\rightarrow$.

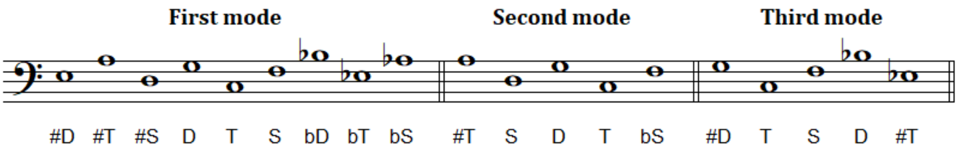

Figure 5: Some notes are assigned different functions depending on the governing mode. Within each measure, every transition in the figure corresponds to $\downarrow$.

Given a musical material to be discussed, once we have established its general tonal contour, writing down the dynamics of the fundamental bass is relatively straightforward. In order to realize the potential of this model of analysis, we must argue precisely which modes can correspond to each passage and contemplate the interaction between real and fundamental bass, as well as that between structural modes and general harmony. We denote the current mode by $X_m^{\flat/\sharp n}$, where X is the fundamental note, m is the structural mode, and $\flat/\sharp n$ refers to the number of alterations which the correspondent diatonic mode has related to the major scale (e.g. $F_2^{\flat 2}$ signifies a second structural mode based on F Dorian). An early interpretation of Bill Evans' *Blue in green* could appear to be like the following:[4]

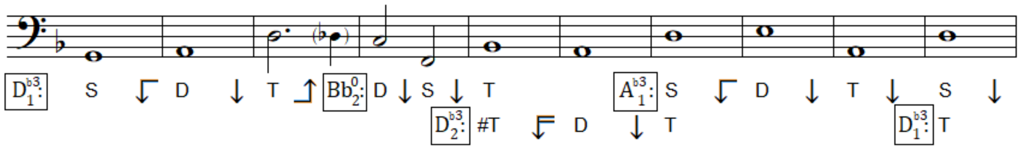

Figure 6: *Blue in green* presents some chords which can theoretically be explained through modulation, but it is convenient to write down any possible double function in order to portray their simultaneous character as chords within the original tonality of D Aeolian.

In the next chapter, we will explore the role and limitations related to the circle of fifths (known about since [Han48]'s work), which presents a challenge in trying to unify a sequence which might contain a diminished fifth instead. On a similar note, one of the difficulties resides in the formation of *double-stars*, i.e. concatenated –usually second– structural modes which embed a mode into a diatonic tonality. This involves some background on voice-leading and heavily depends on the context of the remaining musical elements.

1.3 A syntactic approach

Context-free grammars grant the systems of chord analysis the possibility to reproduce a particular set of rules in virtually any layer within the harmonic architecture of the piece. By acknowledging chords or scale degrees as candidates for both surface-level elements and modal regions in a larger structure, we are capable of applying the same sort of logic universally.

Prior to the implementation, these tools require a brief formalization. As a gen-

[4]The chord upon D♭ could correspond to a tritone substitution, but we will address this topic in the subsequent chapters.

erative system, the purpose is to recreate a chord sequence from a single starting point: this is hardly ever considered to be anything different than the first degree of the global tonal center. From here, we visualize the remaining scale degrees as variables which represent the possible chords in the context of a mode, also taking into account secondary dominants and other preparatory chords. A collection of rules is introduced to operate these elements and ultimately convert every leftover degree into nominal chords.

For example, the turnaround sequence $C^{\Delta 7}$, A^{-7}, D^{-7}, G^{7}, $C^{\Delta 7}$ can be constructed from a first degree of C major through the recursive application of the rule which appends a diatonic fifth on the left side, $X \to \Delta/X\ X$:[5]

$$I \to I \;\; I \to I \;\; \Delta/I \;\; I = I \;\; V \;\; I \to I \;\; \Delta/V \;\; V \;\; I = I \;\; ii \;\; V \;\; I \to I \;\; vi \;\; ii \;\; V \;\; I$$

Formally, a context-free grammar is defined as follows by [Mar21]. Let V be a set of *variables* which will model the scale degrees, and Σ (the *alphabet*) a set of surface-level chords represented by their conventional symbols, called *terminals*. Consider $S \in V$ a *starting variable* and P a collection of *production rules* which transform individual variables into strings of elements from $V \cup \Sigma$. Then, $G = (V, \Sigma, P, S)$ is called a *context-free grammar*. If a word (that is, a sequence of variables and/or terminals) y is generated from a word x by applying a finite amount of production rules of G, we say $x \Rightarrow_G^* y$, and we call the *language* of G the set of words z such that $S \Rightarrow_G^* z$.

[Roh20] contemplates two kinds of rules referred to functional harmony: *prolongational* relations (which explicitly reiterate the same degree as $X \to X\ \ X$) and *preparatory* productions, which anticipate the arrival to a certain chord by adding another one at its left-hand side, based on typical progressions from the tonal tradition. The combination of the two types of rules yields a wide array of possibilities, given that the several instances of prolonged degrees can be prepared by independent flows of chords. This offers a potent mechanism to recreate complex sequences,

[5] Take into account the arrow in this system bears no relation to the structural bass, as it stands for "chord X generates/becomes chords Δ/X and X.

but in this potential also reside some conceivable flaws regarding the continuity of the resulting word.

Below are listed the main preparatory rules in the content of classic jazz standards. A brief look at a selection of tunes should be enough to understand the importance of –possibly secondary– dominants and flat-wise progressions in the sense of structural modes, which explains the abundance of such kind of rules among the entirety. Likewise, the approach to dominant-related chords is contemplated through a series of productions specific to the fifth degree.[6] It is essential to understand that the listed mappings usually operate at distinct levels of complexity, meaning that some of them (such as the diminished passages and approximations) are more bound to appear near the surface chords than others like plagal preparations.

So far, every rule has had as an output a juxtaposition of a degree and the original variable. Nevertheless, there exist special cases which allow us to handle certain harmonic digressions. One can take an arbitrary degree and replace it with a substitution, as in the case of a tritone: we represent these rules, for instance, as $X \rightarrow \flat V/X$. This opens up the possibility to pivot between modes in particular places of a harmonic outline, so –for that purpose– we refine our notation to include our current tonality as a subscript of the degree ($X_{key=Y}$).

If we decide to regard a chord as a first degree of another mode, once we derive it as some X of the original key Y (thus "connecting" it to the higher levels of the structure), we will apply the rule $X_{key=Y} \rightarrow I_{key=X/Y}$. These modulations can be stacked and we will usually implement the rule wherever it is more consistent with the perceptual notion of a tonality change, as it would occur in the case of a turnaround or a simple dominant chord. Similarly, we can analyze backdoor dominants and tritone substitutions of the dominant by establishing, respectively, the degrees $\flat III$ and $\flat V$ as the new, provisional tonic. Lastly, the notation $inv(X)$ can be used to borrow chords from the inverted mode (minor in the case of major,

[6] In the sense that the rule does not apply to arbitrary scale degrees X, but rather only to V.

and vice versa) as long as the note which corresponds to the degree is common to both modes.

Diatonic fifth	$X \rightarrow \Delta/X\ \ X$	Substitution	$X \rightarrow Sub/X$
2^{ry} dominant	$X \rightarrow V/X\ \ X$		
Leading tone	$X \rightarrow vii^{\circ}/X\ \ X$	Tonicization	$X_{key=Y} \rightarrow I_{key=X/Y}$
Half cadence 1	$V \rightarrow \flat VI\ \ V$		
Half cadence 2	$V \rightarrow IV\ \ V$	Modulation	$Z/X_{key=Y} \rightarrow Z_{key=X/Y}$
Diminished 1	$X \rightarrow X^{\circ}\ \ X$	Backdoor V	$V/X_{key=Y} \rightarrow V_{key=\flat V/X/Y}$
Diminished 2	$X \rightarrow \flat ii^{\circ}/X\ \ X$	Tritone subs.	$V/X_{key=Y} \rightarrow V_{key=\flat III/X/Y}$
Half-diminished	$X \rightarrow ii^{7\flat 5}/X\ \ X$	Mode inversion	$X_{key=Y} \rightarrow X_{key=inv(Y)}$
Plagal cadence	$I \rightarrow IV\ \ I$	Terminal rules	$X \rightarrow \text{Chord}(X)$

Table 1.1: Summary of the possible rules for a generative syntactic model of harmonic sequences, in the context of jazz standards.

Some of the previous formulas only –or especially– apply to certain values of X or Z.[7] Whereas we would find instances of $I^{\circ}\ \ I$ recurrently, the succession $\flat ii^{\circ}/X\ \ X$ is prominently more frequent upon $X = ii$. In the context of jazz standards it is usually sufficient to assume $Z \in \{I, V\}$ (or, scarcely ever, $Z = II$) for modulations. However, the tonicization rule falls to a redundancy when we consider the new tonic to be $I_{key=Y}$. In the following chapters we will invoke simple substitutions by replacing Sub with the appropriate degree.

When it comes to unfolding the earlier collection of expressions into a full-fledged syntactic analysis of a chord sequence, we usually start with an initial variable $S = I$ and apply the pertinent rules until an equivalent string in terms of scale degrees is achieved. It is common practice to represent the process as a tree graph,

[7]Formally, the expressions with X, Y or Z actually stand for a set of rules where each of the previous letters is substituted with a particular variable, that is, a scale degree. The expressions are not to be understood as rules, because context-free grammars can only replace one variable at a time. Otherwise, given an expression featuring two X (or variables which depend on X), this would imply that X could be replaced with different degrees: for example, if X was used as a placeholder for any scale degree, the secondary dominant rule could be corruptible by being applied sooner than the degree replacement ($X \rightarrow V/X\ \ X \rightarrow V/II\ \ III$), rather than afterwards ($X \rightarrow III \rightarrow V/III\ \ III$. This argument does not concern superscript notations, since they are often regarded as supplementary information.

so that the decisions made are reflected in a more distinctive manner and the relations between harmonies can be observed graphically. The following is a possible generation of the harmony in the aforesaid *Blue in green*:

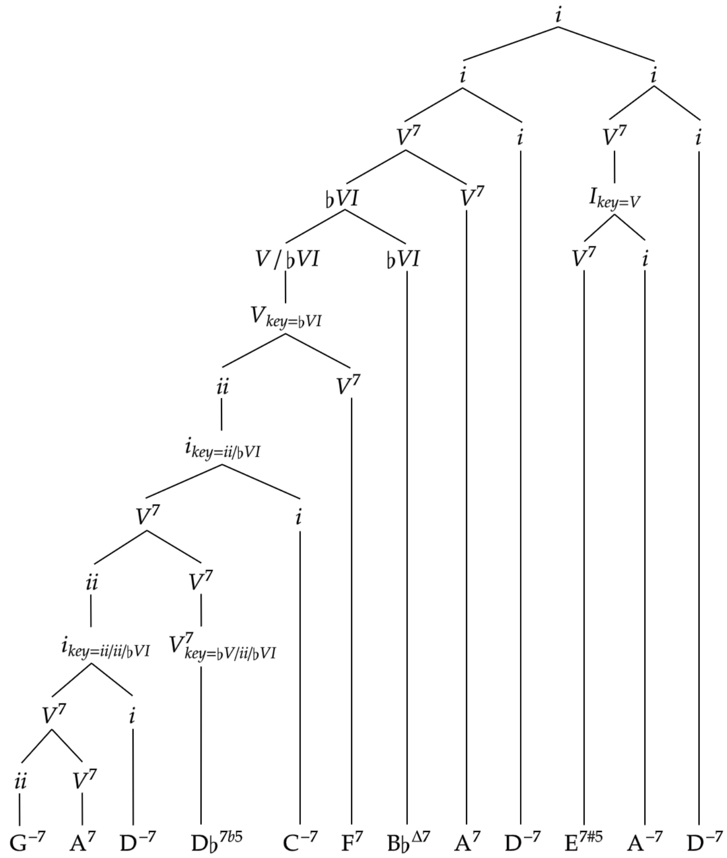

Figure 7: This analysis of *Blue in green* focuses on the right-headed drive of the harmony, since the first instance of D^{-7} could have been thought of as an independent branch overall. Thus, the "change" and the tritone substitution are integrated within the general outline. Note that the *key* notation is dropped for a neater result.

The ramifications of such a powerful tool will lead to opposing interpretations of similar phenomena, and seemingly unorthodox solutions are likely to occur while trying to argue the coexistence of different modes in a same musical piece. These situations will be detailed in chapter 2 at the same time that some common exceptions are presented and assessed.

1.4 Automata and grammars

Formal grammars are highly valued due to their tractability and their relation with algorithms. The computability of some processes allows for specific kinds of grammars and coded programs being used interchangeably. Essentially, both grammars and *automata* deal with the identification of words which can be derived from a collection of rules or instructions, so by studying the affinity between the two, we will be capable of processing harmonic information in a more precise way, or even distinguishing chord sequences based on their inner logic.

Let us begin by providing an informal conceptualization of an automaton, following [LP98]. Given a word, a string of characters from an *alphabet*, we can read it from left to right to decide if it belongs to a particular language. For that purpose, let us consider a set of *states*, which we can viewas vertices of a graph, and select one that will serve as a starting point. For every incoming character, we will move from one state to another depending on the functioning on the automaton, which is set and maps every combination of a state and a character to another state. After reading the entire sequence, we will assert that the automaton *accepts* our word if the process ends in one of the *accept states*. The set words accepted by M will be called the *language* of M, or $L(M)$.

Accordingly, we can now define a *deterministic automaton* as a structure $M = (K,\Sigma,\delta,q_0,F)$, where K is the non-empty set of states, Σ is the studied alphabet, $\delta : K \times \Sigma \to K$ (called the *transition function*) is the map from a pair (state, character) into the next character, $q_0 \in K$ is the *start state* and $F \subseteq K$ are the *accept states*. As the name suggests, any word derived from Σ is fully read in a way that every character can only induce exactly a subsequent step in the automaton, which is clearly portrayed in the inherent graphic representation of the machine.

Be it as it may, there exist generalizations and refinements of the concept which can be used to describe certain languages more precisely: for instance, nondeterministic automata remove the restriction of a map which must consider one and only

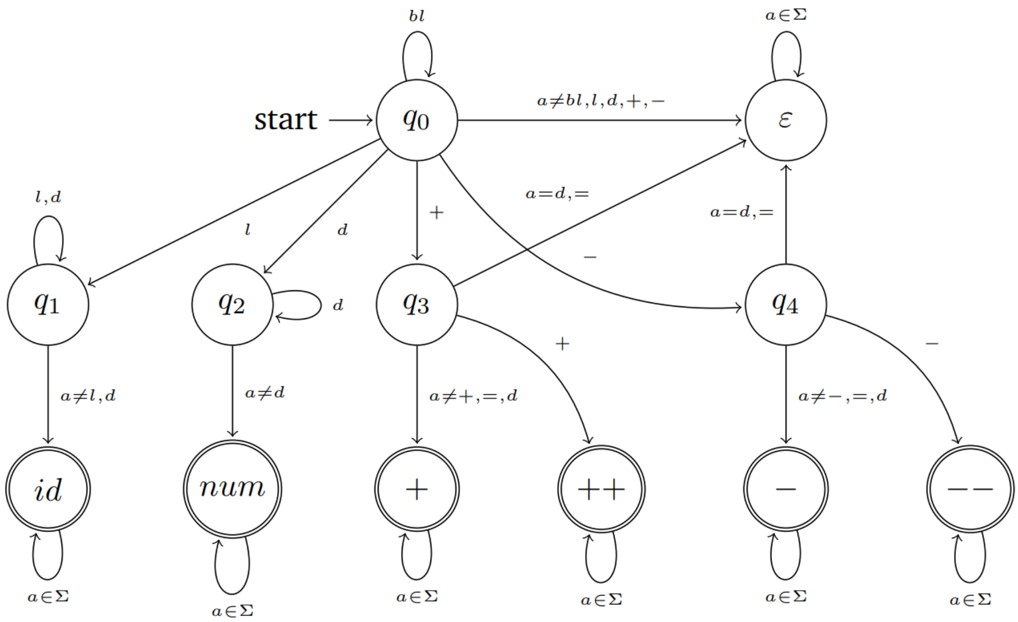

Figure 8: Depiction of a deterministic automaton which recognizes coding variable identifiers, integer numbers, and diverse arithmetic operators. All directed edges indicate what sort of character make the machine transition from one state to another (being *l* "letter" and *d* "digit"). The accept states are highlighted with a double circle and tagged with the corresponding resulting string.

one outcome for any combination, so that the machine can be stuck at a state or it can exit it through various routes. Some other automata print control information or handle several *tapes*, which are the strings of input or output data the algorithms operate. These last types are related to *Turing machines*, which can function back and forth and have been established as one of the most powerful means of algorithm representation.

The interchangeability between certain categories of grammars and automata is proven through several results which involve intermediate definitions and detailed algorithms, as shown in any reference text in theory of computation (such as [LP98]). Building up on the previous definitions, we will now introduce two brief notions and provide some central results which will serve as a basis for the forthcoming explorations.

Let $G = (V,\Sigma,P,S)$ be a context-free grammar. G is called a *regular grammar* if every production rule from P can be expressed as $A \to xB$ or $A \to x$, where $A,B \in V$ are variables and $x \in \Sigma^*$ is a word. A *pushdown automaton* stems from the nondeterministic ones and includes an extra tape for the *stack*, and it is defined analogously by $M = (K,\Sigma,\Gamma,\Delta,q_0,F)$, where Γ is the alphabet of the stack and Δ is the set of transitions (which are of the form $((p,a,b),(q,x))$, where $p,q \in K$, $a \in \Sigma \cup \{\lambda\}$, $b \in \Gamma \cup \{\lambda\}$ and $x \in \Gamma^*$).[8] The following hold [Mar21]:

a) Given a language L, there exists a regular grammar such that its language is L if, and only if, there exists a finite deterministic automaton which accepts L if, and only if, so does a nondeterministic automaton.

b) Given a language L, there exists a context-free grammar such that its language is L if, and only if, there exists a pushdown automaton which accepts L.

Thus far, we have avoided describing in full length how these constructions work in benefit of a lighter reading. This situation will be redressed mainly in chapter 4, where we will propose models based on the presented concepts together with some explanatory examples. In chapter 3, we also mention and illustrate a special kind of grammar which could be useful to develop a unifying system, called *attribute grammar*. It is advised to refer to some of the reference sources should the musical prototypes fall short in their didactic purpose.

[8] λ represents an empty word, and the asterisk expresses finite –possibly empty– words built from an alphabet.

Chapter 2

Analysis of the models

Once the main subjects of study have been introduced, we can discuss their features and their efficiency with respect to the corpus of musical works we set out to explore. This chapter addresses the following challenges: first, why is it –indeed– that we can study the planned tunes, and to what extent can we do so? And, more importantly, how do the initial assumptions from the models shape our analysis of said music? Can we foresee the possible flaws which the systems present? The ensuing observations will allow for a more aware breakdown of the harmonic elements and will provide valuable insight for an eventual design of a model which stems from some of the current ones.

For the sake of succinctness, most of the tunes mentioned in the upcoming pages are fully analyzed in chapter 3. Even though a slightly extended version of the models may be used, the foundations remain unaltered and the applicable conclusions can be drawn for the discussion in this chapter.

2.1 Scope of application

It is made clear that the authors of both records of papers had established their respective proposals upon a collection of possible works to study. This is specially true in the case of [Roh20], who argues the plausibility of the production rules in terms of the elemental harmonic relations recognized in jazz standards. Nevertheless, we can backtrack and attempt to deduce the core aspects of a musical material which abode by the principles of the models.

In order to search a language where structural modes and generative syntax are consistent, we begin by inquiring about the conception of harmony at a higher level: both systems deal with modality in a more or less explicit way, but can the same be stated about tonality? In any of the cases, there are resources to effectively manage harmony as a succession of chords within a tonal region. However, structural modes unequivocally assign functional labels to scale degrees and, even though some workarounds have been proposed by the authors,[9] the model slightly falters when trying to handle –in terms of tonality– such extra-modal occurrences exclusively by means of the bare axiomatization. In that sense, generative grammars are more flexible by dispensing with strict rules which relate functions and chord progressions. Despite that, most of the information respecting tonal functions can be retrieved and interpreted from the nature of most of the production rules (e.g. $\flat VI - V$ works as a subdominant–dominant pair).

On the other hand, the dynamics of the structural bass offer a more robust method to describe an arbitrary harmonic sequence, since they do not require a tonal signification to realize a movement. Regardless, the combination of arrows provides some direct understanding of the position of the bass within the mode and the diatonic circle of fifths, as well as of the modulations that may have taken place. For instance, performing a tritone substitution of the dominant in $ii - V - I$, we can represent the sequence by $(\Leftarrow\downarrow, \Rightarrow\downarrow)$, which preserves the nature of the circle of fifths through the presence of down arrows, and undoes the substitution by cancelling out the horizontal arrows. This knowledge is not as immediate in the case of our syntactic model: notably when it comes to chord changes (as seen in *Stella by starlight*, figure 17), there are not standard rules which can replicate some of the gaps. Usually the analysis is faced with the decision to either integrate the change

[9]This improvement and some others will be examined in the next section.

within a substitution (which can later be contextualized) or to let the unconnected divide express such interruption in the ever-present momentum of chained fifths.

In any case, we can establish that both of the languages suffice to manage tonal (functional) contexts. The potential appearance of a *brief* passage with improper harmony could nevertheless be shoehorned into the models as long as it might be understood as an exception within the formalization. Any proposal to parse a progression such as $I - ii - iii$ is better understood as an instance of a higher formal layer: this is the case for the first bars of *My foolish heart* (figure 18), where a sequence of juxtaposed relative $ii/X - V/X - i/X$ cycle through the first degrees and then returns to the tonic backwards (i.e. $I - ii - iii - ii - I$).

Plagal production rules are not to be overlooked, as they play a remarkable role in arguably one of the most popular chord structures in jazz: the *12-bar blues form*. While it is mostly identified by the presence of a IV^7 degree chord in measures 5 and 6, and a cadential sequence during the last four bars (usually $V^7 - IV^7 - I^7 - V^7$), [Roh20] and others also suggest the possibility of accepting as relatives to the 12-bar blues some harmonies which involve such $I - IV - I$ inflection at a higher formal level. For example, Charlie Parker's *Blues for Alice* features a IV^7 chord in the fifth measure which could be interpreted either as the $IV - I$ division of the final I chord or as the secondary dominant of a subsequent $\flat VII$ degree. The first option would strengthen the connection between the tune and a structural vision of the 12-bar blues.

Choosing jazz standards as the subject of analysis appears suitable enough considering the distribution of transformations from each system: in embedded structural modes, diatonic fifth-based motion serves as the main resource, whereas structural augmented prime alteration helps reposition the stream of circles of fifths. This could leave candidates such as Italian Baroque music, but the specificity of some syntactic rules such as backdoor dominants and other substitutions does not match their common practice. The conception of our grammar as a model based on fifths holds up, taking into account that rules which are unique to a particular scale degree (with the exception of plagal cadences) are either based on dominants or founded on diminished chords, which often function as leading chords and are not substantial enough in the grand scheme of things.

2.2 Features and implications of the models

Since most of the examples which we discuss below are presented and fully analyzed in the following chapter, let us begin by demonstrating some interpretations of the standard *Autumn leaves*. At first sight it is trivial to determine the ruling dynamics of the chord sequence: a circle of fifths. It is the interaction between harmonic rhythm and melodic structure what raises an unexpected amount of difficulties. When parting from the examination of the bass motion, structural modes of C and A can be deployed in a symmetrical manner to match the melodic contour – ignoring the $F^{\Delta 7}$ (which will be figured out following [DN19] during the next 3 subsections), the ensuing figure solidly summarizes the different musical elements in play:

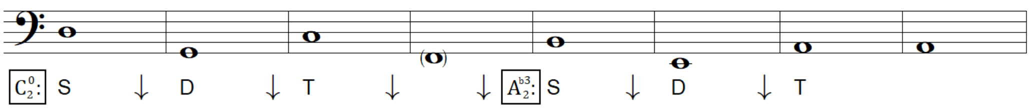

Figure 9: Due to the 4+4 structure of *Autumn leaves*, the strong presence of ii-V-I progressions causes the $F^{\Delta 7}$ adopt a bridging purpose rather than its anticipated tonal function.

However, the lack of a binding assignation of tonal functions in the syntactical approach allows for less specific interpretations to prevail. We can either parse the chord sequence as a plain chain of diatonic fifths or impose a modulation to the relative key and even consider the $F^{\Delta 7}$ occurrence to be a preparation of the dominant. In this sense, the strictness of structural modes leads to a more coherent depiction of the general form and the behavior circles of fifths (and double-stars), right from the beginning.

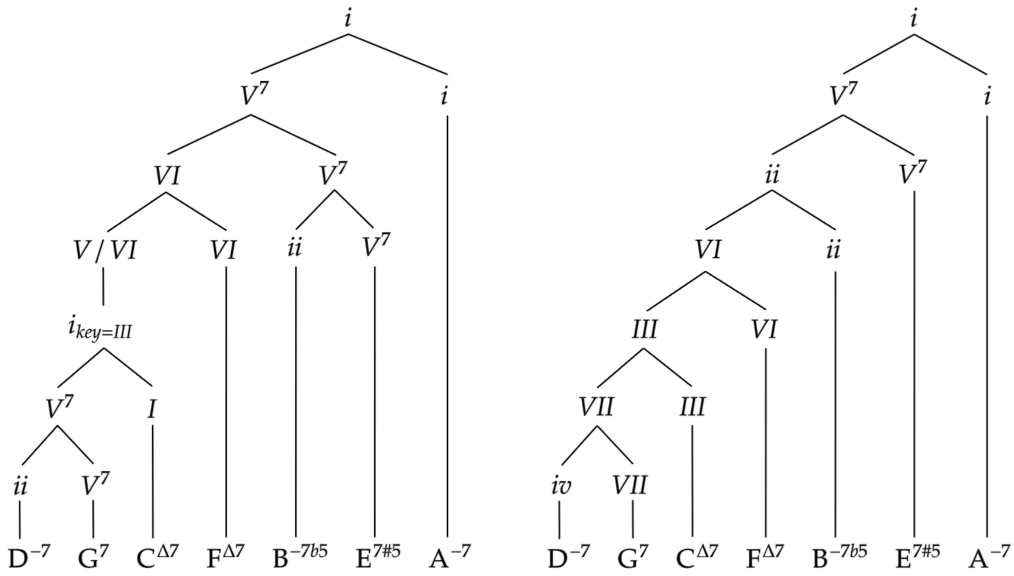

Figure 10: Side-by-side syntactic analyses of *Autumn leaves*: modulating version (left) and monotonal recursive application of the diatonic fifth rule (right).

For the rest of this section, we will break down the suppositions that each model considers about tonal harmony, as well as the main dubious aspects with respect to: form and modes themselves, chord dynamics within tonality, connectedness of harmonic sequences and reiteration of structures.

2.2.1 Core assumptions

Being harmonic systems of analysis, their respective understandings of chord motion are substantiated by central claims. In the case of Rohrmeier's generative grammar, the fundamental principle is claimed to be the momentum of harmony towards the latter end of progressions, which recieves the name of *right-headedness* ([Roh20]). In terms of generative context-free grammars, this automatically translates to *left-branching* rules, meaning that production rules will mostly be of the form $X \to Y\ X$ whenever they map to more than one symbol. In that sense, chord dynamics work as the resolution of tensions portrayed graphically by gaps between branches on the tree representation. This is noticeably present in three occasions:

- Chord changes which partly avoid dominant preparations, leading tone connections and fifth-based chains (arguably the driving force of most standards) at some point, including those disguised under substitutions and borrowings.
- Sequences which omit the tonic at the beginning, and thus characterize their global tonality through the harmonic goal.
- Branching at more primitive levels, which often describes the tonal regions visited along the tune (as seen in the 12-bar blues form) and may span more than a single phrase.

Mainly the last point creates a dialogue between harmonic structure and melodic form, causing overlaps which sometimes explain certain phenomena. On the other hand, the absence of right-branching rules (like those introduced by Lerdahl and Jackendoff's Generative Theory of Tonal Music) does not compromise the ability to generate progressions, and actually eases some of the restrictions imposed by departure-resolution patterns.

To illustrate the kind of changes which build up tension, *My foolish heart* begins with a temporarily modulates to the degrees *ii* and *iii* by jumping from the last chord of the previous mode (I or ii) to ii/ii or ii/iii, in order to establish a relative $ii - V - i$: this accumulated tension is resolved by descending back to *ii* and I chaining relative progressions of $ii - V - i$, which are now connected along a chain of fifths until the original tonic is reached. The symmetric nature of such tension-deflation structure is reflected graphically (as clearly seen in figure 18) by the left-branching generatedness of the syntax, and it is reinforced by the fact that the discontinuities are usually made recognizable through the variation of alterations with respect to the main key.

As we established in section 1.2 the analysis based on structural modes stems from the motion of the fundamental bass. The enclosure of modes is then defined by their three pillar notes, provided that their perception within the tune matches the tonal function which they inherently have assigned. Even though this induces a clear differentiation between the modulating and the progressive character of the bass, the role of the diatonic diminished fifth and the gray areas between modes still maintain some level of ambiguity which needs to be solved beyond the initial formalization of the model.

2.2.2 Structure and tonality

A priori, structural modes offer a description of the tonal regions but do not include an inherent logic to analyze the form, understood as the hierarchies and relations between such regions. The syntactical approach does, but the allocation of tonalities and boundaries is often neither straightforward nor explicit, meaning that many decisions (like applying a tonicization or continuing on using recursive dominants V/X) are left to the judgment of the author of the analysis. Therefore, rather infer what they understand about the form through the branching decisions and small liberties that the model allows for. For instance, again in *My foolish heart*, the second chord ($E\flat^{\Delta 7}$) may be thought of as a IV degree from $B\flat$ major or as a preparation for the following D^{-7}.

Additionally, it is common to stumble upon examples where the definition of the confines is paramount to the formal analysis. In *Stella by starlight*, we will ultimately consider the chords $E^{-7\flat 5}$ as the leftmost elements of their respective branches, thus comprehending the piece as a series of goal-driven resolutions of $E^{-7\flat 5}$: these are underplayed by the immediate appearance of another instance of the chord followed by its string of resolving harmony, until the final tonic is reached.

However, this analysis poses two main challenges. First, the melodic structure (supported by the separation of the lyrics in two verses) does not coincide with our harmonic form, because it splits the central branch at the chord G^7. On the other hand, interpreting the modulation in $E^{-7\flat 5}-A^7-C^{-7}-F^7$ not as a disruption, but rather as a tonicization within the branch, raises the subsequent uncertainty: if such a change is always unified, why are all the occurrences of $E^{-7\flat 5}$ detached from its previous chord? While we already accounted for a disagreement between the two structural perspectives, it would suffice to justify the second problem for us to admit the first as a feature of the standard, instead of a flaw from the formalization.

We will undertake this issue in section 3.1 after introducing an adjustment to the model of structural modes, which will allow us to handle sequences as an instance of an individual chord. This will accentuate the importance of structural alterations ($\leftarrow$, $\rightarrow$) as formal resources and will be useful to formalize reasonings related to double-stars within circles of fifths, together with a hint to the equivalence between some concatenations of arrow transitions.

2.2.3 Intratonal relations

Through the formalization its underlying foundations, the role of the circle of fifths has been established as one of the central means of chord progression within a tonality, if not the most relevant one. Recently, we considered the possibility of breaking a fifth-related pair of degrees into two different components of a tune, but one can also question if the opposite could hold, i.e., to what extent do circles of fifths underlie our harmonic sequences?

A strong case for embedded structural modes comes from the analysis of the second 4-bar fragment in *There will never be another you*. Backed by the sudden deceleration of the harmonic rhythm, the C^{-7} chord seems to be occupying what could have been a $C^{-7}-F^{-7}$ sequence[10] (which would have smoothened the entire progression into a circle of fifths). Instead, it skips directly to $B\flat^{-7}$ through the combination of a flat-ward alteration ($\rightarrow$) and a sharp-ward progression ($\uparrow$), which can actually be decomposed into

$$[\rightarrow\uparrow] \cong [(\downarrow\downarrow\downarrow)\uparrow] \cong [\downarrow\downarrow].$$

This indeed resonates with the absence of a fifth, and it is further supported by the fact that a hypothetical modulation $\rightarrow\uparrow$ towards the second mode of A$\flat$ major would endow C^{-7} with a sharp-ward dominant function ($\sharp D$), which needs two flat-ward fifths to reach the function S of $B\flat^{-7}$.

[10] Some recordings and versions of its lead sheet do include the intermediate F^{-7} chord.

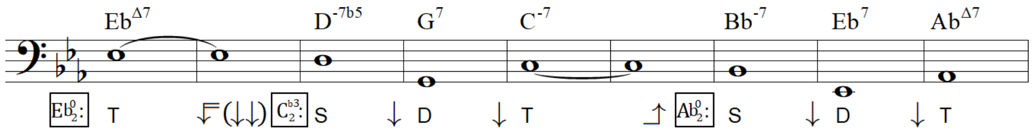

Figure 11: The C^{-7} chord in *There will never be another you* perceptually hides a flatward fifth progression.

The double nature of these changes can not seemingly be reflected by a syntactical analysis, and it would even require a less straightforward rule usage to express the basic progression, unlike a previous formalization which [Roh11] provided with an identifier of tonal regions. Analogously, one could conjecture whether the second measure of the standard could conceal a fifth-related chord $A\flat^{\Delta 7}$. Yet this initiates the discussion of the role diminished fifths occupy in the investigation of structural modes.

In the vertical dimension, half-diminished chords are frequently found jazz standards, even in as prominently as in *Round midnight* (see figure 15). But, intervalically, a keen eye will have probably noticed how the interpretation of *Autumn leaves* (figure 9). [DN19] substantiate the generic and specific embeddings of structural modes within full diatonic tonality, and the latter distills the capability of flat-ward progressions ($\downarrow$) to represent diminished fifths as well: the sequence of degrees $\hat{6} - \hat{2} - \hat{5} - \hat{1}$ appears in several corresponding cadences from Chopin's Preulde Op. 28 No. 20, although with varying –diatonic– modes each time. This causes the bass to possess different alterations and the intervalic diminished fifth to switch positions, even though the structural mode and its tonal functions remain unaltered. This argument is used to allow for the motion of fifths to lack a semitone in certain contexts, while still being represented by $\downarrow$.

Having assimilated this detail into the model of the structural bass, it is possible to parse diatonic circles of fifths in a more regular manner, similar to that in the syntactical system. Recalling the case of figure 9, we can combine the newfound smoothness of structural modes with *non-structural progressions*, which deem a degree and its surrounding transitions external to the contiguous modes. With that, double-stars generated by circles of fifths are completely well founded, and structural modes prove to be more more effective in that sense.

2.2.4 Contiguity, disruption and repetition

There is yet another way to contemplate how, even if the leaps within any of the models are part of their expressiveness, sometimes the corresponding gaps between neighboring surface chords are narrower than we expect. As we already discussed, embedded structural modes offer a continuous and integrated view on chord sequences, but in larger structures it ultimately depends on an explicit –written– description to interpret the hierarchies between modes. Taking as an example the motion $B\flat^{\Delta 7}-E^{-7\flat 5}$ in *Stella by starlight*, what could seemingly be represented by the production rule $X \rightarrow \Delta/X \;\; X$ is instead left as a divide so that the syntax of right-headed, left-branching harmony allows us to parse it as a new tension build-up. It could nevertheless appear as a subordinate structure at the cost of needing an extra means of interpretation, such as the one which we will introduce in section 3.1 for structural modes.

A separate but equally visible situation arises from the half cadence in C major determined by chords (in root position) $F^{\Delta 7}-F\sharp^{\circ}-G^{7}$. Much like in similar cadential scenarios (like the one presented in figure 12), the right-headed gravitation of dominants prevails over any sort of proximity between nearby chords, in a way that $F^{\Delta 7}$ and $F\sharp^{\circ}$ are both presumed to be preparations of G^{7}.

However, the solution based on structural basses appears to be more convoluted when trying to consider F♯ as a structural note. Since the brute force possibility of modulating to E minor seems somewhat unfeasible (it would require the transitions [$\uparrow\leftarrow\leftarrow$] and [$\downarrow\downarrow\rightarrow$]), let us decompose the bassline $F-F\sharp-G$ into simpler moves: by introducing a C between F and F♯ and modulating to the dominant, we can solve the diatonic augmented prime as two instances of a fifth which differ in a semitone due to their embeddings in C major and G major. Our goal is to connect the current F♯

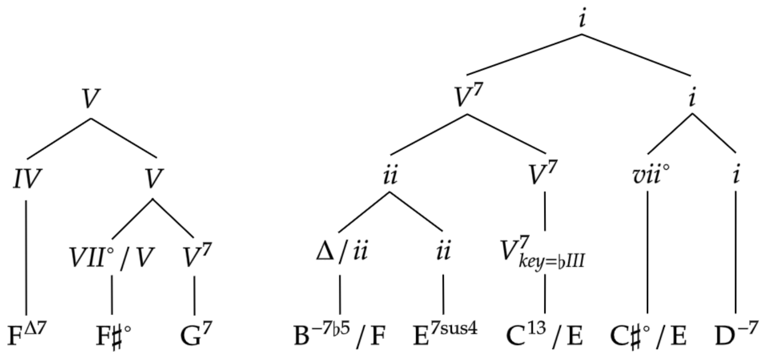

Figure 12: The prominence of dominants causes certain chords to be considered as preparations of the *V* instead of a linear sequence, as shown by the syntactical breakdowns of a half cadence (left) and a conclusive cadence with limited voice-leading distance (right).

with the closing G in the context of a same mode, so we will introduce the sequence F♯−A−D-*G* and apply the transformations →, ⇂ and ⇂. This prompts us to employ a second structural mode of G for the analysis; despite that, we soon come across a conflicting interpretation of the tonal function which F♯ possesses. On the one hand, C (functionally, ♭*S*) progressing flatwards to F♯ causes this degree to be assigned a ♭*D*. Nonetheless, by going backwards from G, it is corresponded with a ♯*S*, which differs from the previous inference.

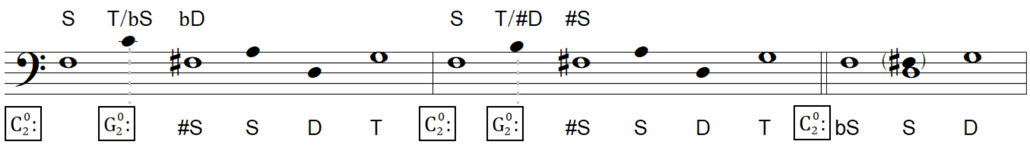

Figure 13: Several interpretations of the half cadence bass F−F♯−G, showing the conflict between the two ends of a circle of fifth (left), a provisional solution which does not provide an authentic description of the first two added intervals (center) and a proposal which reinterprets the fundamental bass.

Even though the source of this incongruity could be tracked back to the usage of a single structural mode for the latter tonal center, the main oversight is having un-bound an approximation of the circle of fifths from a meaningful tonal signification of the diatonic augmented prime. Such a decision has led to the –expectable– outcome where a diatonic 7-note cycle presents a mismatch with a coprime cycle with only 3 notes. As seen in figure 13, the situation could be addressed by employing B instead of C, but the first two intervals are still deprived of tonal meaning in the cadential context.

All things considered, since the chord F♯° has a functional role in the sequence F−F♯−G, there only remains to regard it as an contraction of D^9, so that the transition F−D can be represented by $\flat S \leftarrow S$ in the original key. Note that the diminished fifth within minor keys is not liable to the same complication, and those stemming from a tritone substitution will be easier to parse through the motion of structural basses ($\Leftarrow\Rightarrow$) than with a syntactical approach.

In a broader perspective, whenever we set out to analyze a larger section (or one with a more irregular harmonic outline), one of the main limitations will result in attaining an almost mosaic-like sequence of modes. This is particularly ordinary for tunes like Coltrane's *Giant steps* (figures 20 and 21) or Evans' *Orbit*, and as a consequence the relationship between form and harmony might appear to be atomized or degraded. During the next chapter, we will revisit a remark from [DN18] and some of our previous comments as a starting point to a model which will redress the issue.

On the other hand, Rohrmeier's model also raises questions regarding the organization between structures which symbolize the same harmony or material. As it occurred in other cases, we can decide on whether to place a ramification near the higher constitutional level, or within the leaves of a branch. Therefore, we can emphasize the bond and impulse of left-branching generative harmony, or opt for an analysis which may be identified with the melodic aspect or a traditional form. For instance, we divided the four harmonically connected segments from *Stella by starlight* into two segments: the exposition in the original key, and the juxtaposition of the transposed region, the return to the tonic, and the conclusive reexposition of the theme. In this manner, the structure $[I - [[V - I] - I]]$ depicts the tension-resolution behavior which [Roh20] and others consider crucial in tonal harmony.

2.3 Considerations for a new proposal

We will now suggest some possibilities to create feedback between the models and formalize new versions afterwards:

Regarding structural modes, our main concern is to equip them with a mechanism to forge a more solid interaction with the form of standards. A significant improvement would entail being able to connect some of the successions of modes whose embedding tonality only persist two bars (as in *Stella by starlight* or *Giant steps*). In addition, setting a context where different modes coexisted could explain the usage of certain scales or chord tensions in a certain region. To do so, it could be possible to consider neutralizing combinations of transitions as a way to briefly escape a mode, instead of categorizing a passage as a new tonality (as happened with tritone substitutions). This, indubitably, would need to preserve the aspects considered so far, like double-stars, non-structural progressions and, to some degree, the flexibility around the perimeters of the modes.

In the case of generative grammars, an addition which would not compromise the integrity of the system would be that of a rule for structural augmented primes, although the role of the interval as a possibly modulating device would need to be clarified. In opposition to this idea. A similar proposal would go along the lines of the first interval displayed in figure 11. Contrarily, an idea which could bring our grammar closer to structural modes (as well as making diatonic fifths more navigable) would be the addition of some kind of tonality-marker.

Over the course of the next chapter, we will be capable of identifying features which could set the bases for a mixed model, such as the specificity of grammars or the accuracy of structural basses to define movements and modes.

Chapter 3

Interaction between models

Having presented the specifications and the underlying notions behind each of the models, the current chapter lays out several approaches to enhance or address the systems from the specificity of some aspect which could be considered lacking in the raw formalization. This will serve as a transition from the resources each way of analysis offers, towards the proposal of an amalgamated model.

The first section will introduce and discuss a modification for the analysis of embedded structural modes (hinted in [DN18]) while providing a detailed implementation of both models over some examples, with the objective of illustrating the contents from the previous chapter. Later on, the succeeding section will suggest some variations of grammar-based systems as a link between the theoretical bases of chapter 1 and the following results in chapter 4.

3.1 Recursive structural modes

In the elementary examples used to demonstrate some features of structural modes, it can be seen how the partition of periods consists of resolutions on the tonic of local modes, whether their ends are superimposed or not. These usually involve the degrees *vi* and *I*, which are related by a diatonic minor third, a structural transition which preserves their function up to the addition of a $\sharp$ or a $\flat$. This motion occurs at a higher formal level, induces a cohesion between chords in a same mode and leads us to the possibility of considering such sequences as units.

Similar to the organization of tonicizations and modal regions in the syntax of jazz standards, some blocks of chords can be related to each other based on their representatives. Thus, we may view a harmonic sequence as a juxtaposition of structural modes which summarize the general dynamic of each fragment. However, there are some aspects that require a precise deliberation:

The description of any structural bass motion is subject to the election of a mode which will act in a larger temporal scale. In other words, we need to choose modes which will encompass the proper structural modes as we studied them thus far. This can either result in the apparition of a tonality which represents the entire piece, or a weaker scenario where no "meta-mode" can enclose more than one or two conventional modes. For that purpose, we propose to implement a looser insertion (in a way that, for example, a $\flat III$ degree could belong to a I understood as the indicator of a backdoor dominant) and apply the process *recursively*, so that eventually one single defining mode might remain.

Regardless of the outcome, it is essential to preserve an interpretation which can highlight the distinctive motion of the actual bass and the connection between blocks. For instance, the modulating first section of *My foolish heart* can be summarized as $I - ii - iii - ii - I$, but –even if the transitions can be depicted by an arrow [←↓] and its corresponding inverse [→↑]– the local harmonic sequences only follow a circle of fifths during the latter half. Due to such incompleteness of the reduced outline, it is convenient to maintain the analyses of various complexity levels side by side.

Ultimately, we will employ three layers of modes: the initial one for the proper description of the fundamental bass, the general one to describe the form and the primordial source of the local tonal regions, and an intermediate step to emphasize the modes which act as pillars within each section. Each element will be assigned a tonal function in terms of the encompassing mode, so that –in a similar way to chords themselves– we can view them either as elaborations or digressions (e.g. $I - IV - I$), or as preparations for the subsequent "meta-mode" (e.g. $I - vi - II$; V).

As the practical analysis concerns, it must begin with a usual analysis of the bass in terms of structural modes. Then, we can perform the same process along the bass defined by the resulting modal regions, in order to establish the first of the three aforementioned layers. The second one is derived from the simplification of juxtaposed transitions which can either be canceled with each other (such as regions arisen from tritone substitutions) or stand for passing chords. Lastly, the final layer will represent the whole tonality or relevant high-scale modulations, if they exist, and it will often be subdivided by vertical lines. We will utilize the typical arrow notation to relate the degrees within every level of complexity.

Introductory piece: *Round midnight*

The standard *Round midnight* serves as a basic example to portray the differences between both models and showcase the decision-making process behind some structurally ambiguous harmonies. Let us begin by analyzing the bass motion:

We consider the prevailing tonality to be E♭ minor, although the choice between the first, second and third mode will depend on the local harmony. Parting from a recording or a lead sheet, the foremost task needs to be the differentiation between real bass and fundamental bass: we will only consider the second one, thus the first measure is represented by $\mathrm{E}\flat^{-7}$. The one-to-one reexposition of the first eight measures is simplified and depicted as a final tonic chord.

We decide to adopt the mode $\mathrm{E}\flat_3^{\flat 3}$ for the first three measures, so that the backdoor $ii - V$ ($\mathrm{A}\flat^{-7}$–$\mathrm{D}\flat^{7}$) is integrated in the main tonality. This can be supported by the fact that its deceptive resolution to $\mathrm{C}^{-7\flat 5}$ actually uses the same scale as the degree i (C locrian 9, or E melodic minor) and is sometimes interpreted as an inversion of $\mathrm{E}\flat^{-6}$ ($\mathrm{E}\flat^{-}/\mathrm{C}$). In general terms, the rest of the tune develops over an $\mathrm{E}\flat_2^{\flat 3}$ so that F^{-7} (appearing in the various $ii - V - i$) can be parsed as a proper main subdominant.

Nevertheless, bar 4 introduces a rapid sequence which resolves to $A\flat^{-7}$ and acts as its dominant: the choice of an $A\flat^0_2$ mode for the latter two beats is suitable to highlight the relevance of the modulation. It is preceded by $B^{-7}{-}E^7$, which can be interpreted as a tritone substitution of $F^{-7}{-}B\flat^7$ – the $ii - V$ to E♭, the dominant of the target $A\flat^{-7}$. Such a reading unifies the progression in measures 3-5 as a modified circle of fifths, since $C^{-7\flat 5}$ is the diatonic fifth of F^{-7}, the tritone substitiion of B^{-7}.

During the second section of the standard, we use the combination $[\Leftarrow\downarrow] - [\Rightarrow\downarrow]$ to express local tritone substitutions, which can be seen as a resulting $\downarrow - \downarrow$ canceling the opposing structural alterations $\Leftarrow, \Rightarrow$. As seen in figure 11, other ways to convey motion along the circles of fifths appear in the form of $[\leftarrow\downarrow]$ as $\uparrow\uparrow$, and $\rightarrow\uparrow$ as $\downarrow\downarrow$.

Regarding the form, the modulation to *iv* ($A\flat^0_1$) and the final backdoor dominant ($G\flat^0_1$) prevail in the second layer of analysis, but the eventual resolution to the first degree and the accidentals used cause the dissipation of the dominant as the perceptual driving force of the latter half (especially in measures 9-12). We recur to the syntactic approach to redress this misrepresentation:

After invoking the prolongation rule, the second instance of *i* is prepared with a V^7 which comprehends the second section. Moreover, the V^7 is prolonged to represent every half cadence. Notice that the one which contains the $D\flat^7$ chord – unlike before– is not connected to the $C\flat^7$ by a same branch or tonality: instead, the latter is interpreted as a preparation[11] of the last occurrence of a dominant.

Another ambiguous preparation of the dominant (in this case, ii^7/V) takes place at the $C^{-7\flat 5}$ chord in measure 3: following its status in the analysis of structural modes and its morphology, it is considered an object from the original key which is directly related to the dominant. However, it could be linked to the subsequent B^{-7} chord by reversing (applying again) the tritone substitution rule, creating a –grammatically– more solid harmonic progression during bars 3 and 6. The choice between either of the formalizations should be substantiated by the acoustic sensation of proximity in the context of tension-resolution musical structures.

[11] Although the tritone substitution of V^7/V was used, the analogous $\flat VI$ could function as well.

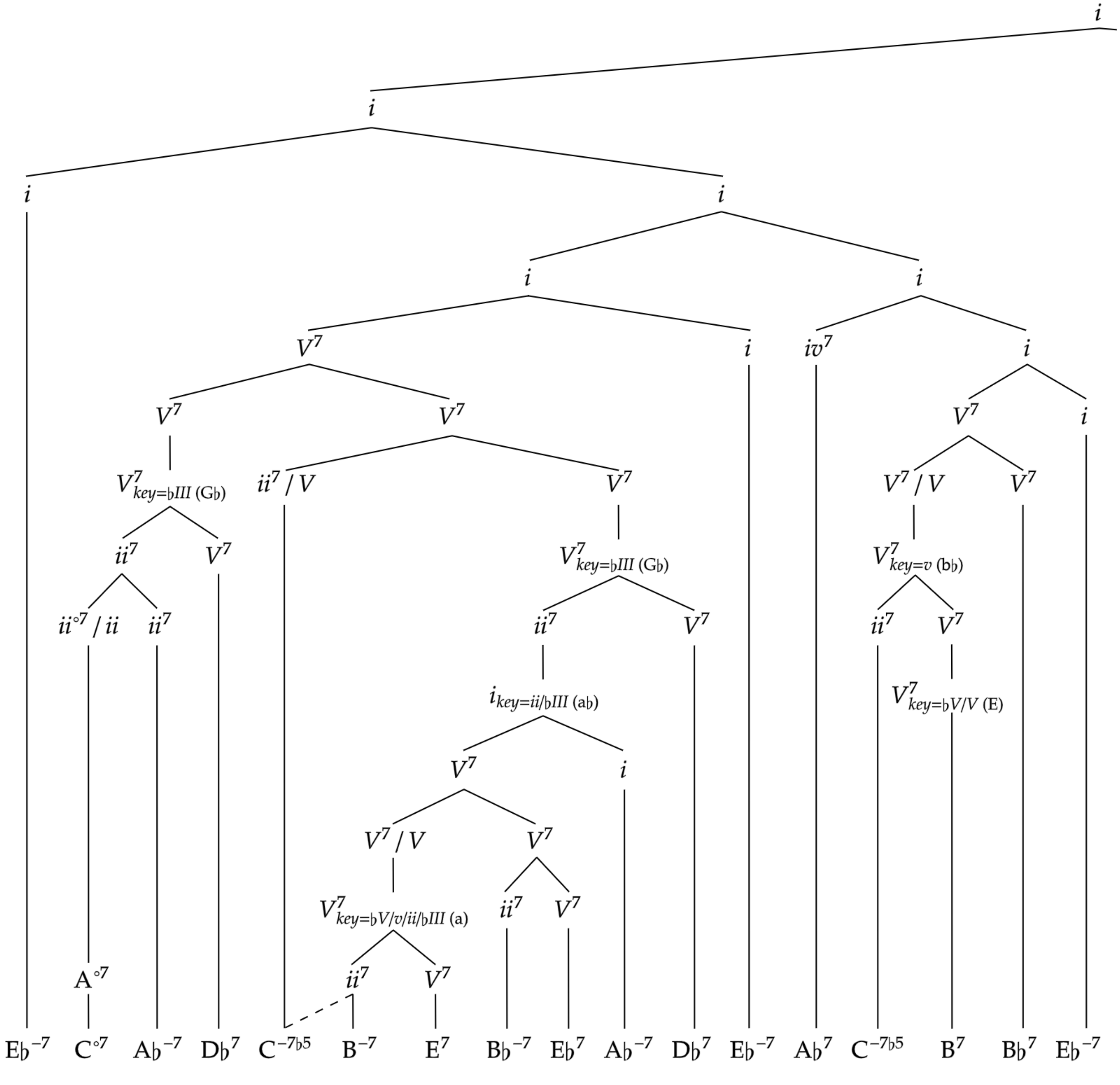

Figure 14: Syntactic analysis of *Round midnight* (left). The dashed line indicates the previously discussed possibility of a tritone substitution.

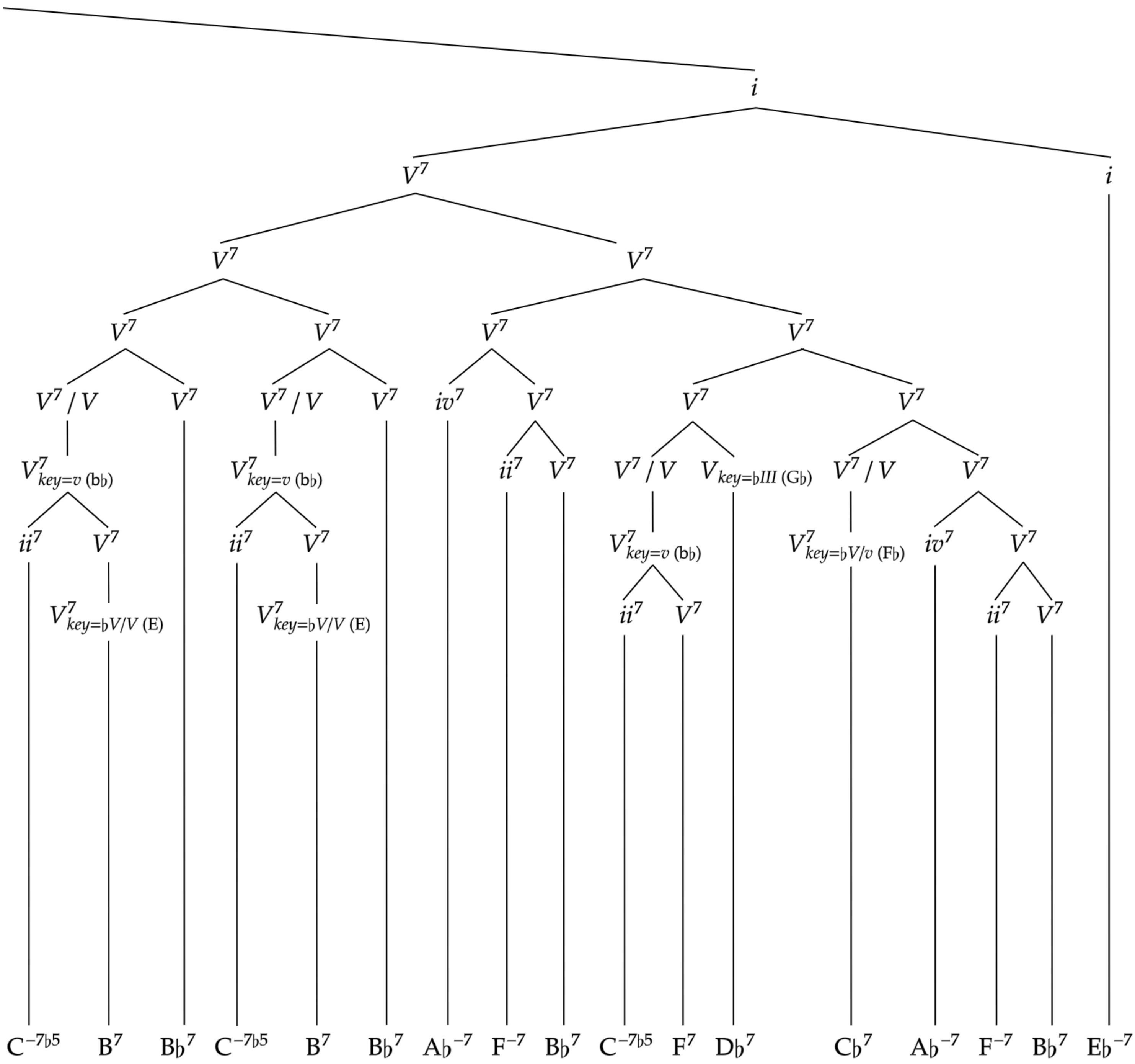

Figure 14: Syntactic analysis of *Round midnight* (right). Notice that the key subindex notation is implicit within branches for the sake of simplicity.

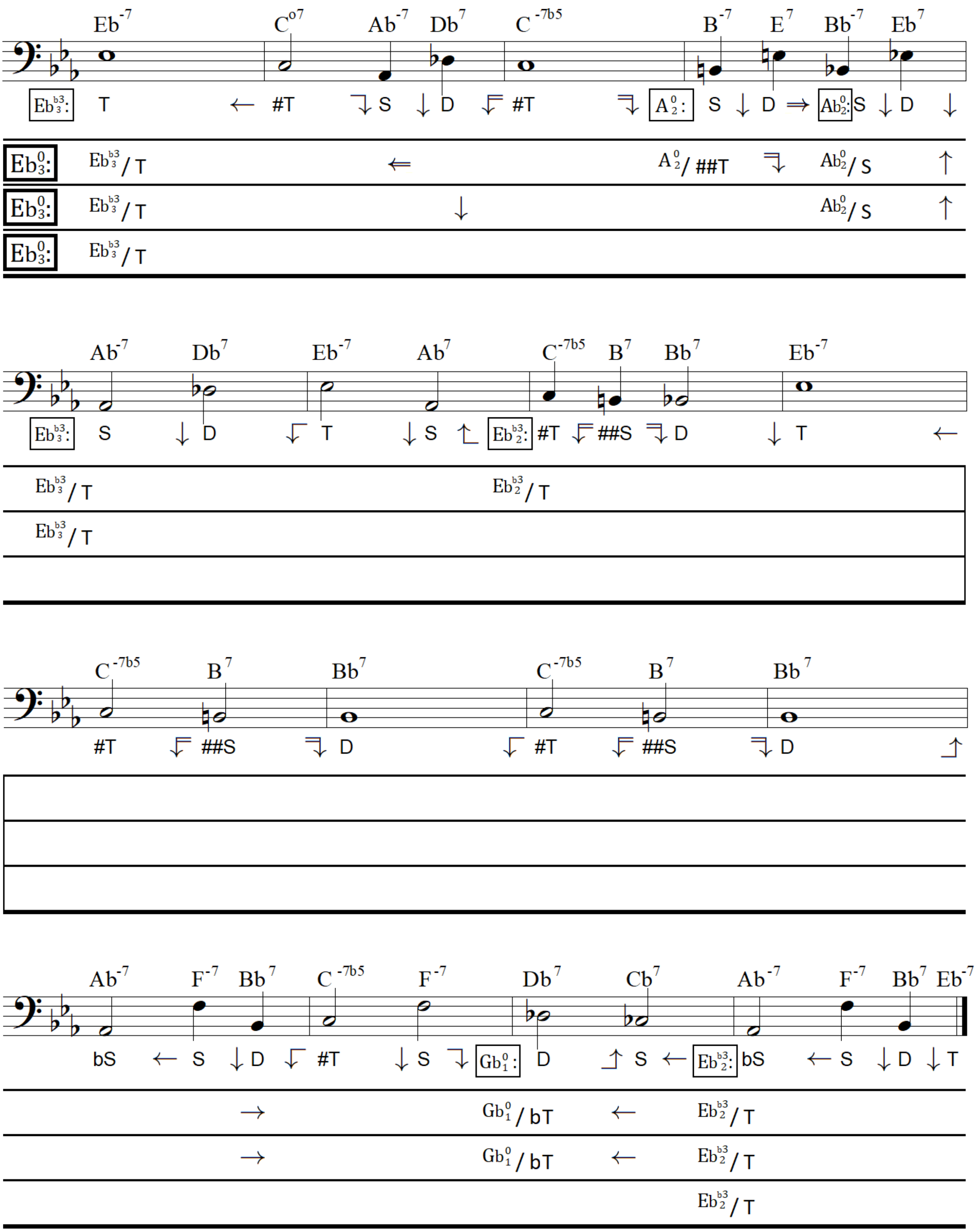

Figure 15: Analysis of *Round midnight* with recursive structural modes.

Changes and expectations: *Stella by starlight*

In general, the presence of major seventh chords helps us determine a first re-expositive structure, which in the subject of formal grammars translates into a sequence $(I) - (V - I) - (I)$. Nevertheless, the melodic/lyrical structure is often regarded as bipartite (16+16 bars), so it is crucial to back our inferred claim in a more detailed manner to justify the disparity. For each of the degrees stated above to constitute a branch, their harmonic progressions must be coherent and possess some degree of right-headed momentum.

Most of the chords within each branch are related by fifths with their adjacent harmony, taking into account backdoor dominants. However, every progression is led by an opening $\mathrm{E}^{-7b5}{-}\mathrm{A}^{7}$ (a $ii - V$ over the third degree of B♭ major), with a sudden change which breaks the underlying circle of fifths.[12] One could argue the possibility of linking these degrees to their preceding harmony, but this is not formally sound due to their suspensive nature and the fact that they are preceded by conclusive chords. Otherwise, if we observe that the last instance of $\mathrm{E}^{-7b5}{-}\mathrm{A}^{7}$ is followed by the proper circle of fifths until the original tonic, one could propose a Schenkerian-like solution to delay the resolution of the harmony up to the ending of the tune. That is just as implausible, but it will prove useful for the approach with structural modes.

For now, let us integrate the change $\mathrm{E}^{-7b5}{-}\mathrm{A}^{7}{-}\mathrm{C}^{-7}{-}\mathrm{F}^{7}$ in the same branch, adhering to the following criteria: the formal modulation is performed over the vi degree of V/F. Not only is vi the remaining of the 4 substitutions often considered for semitone-tone scales on dominant chords (I/V is the dominant, $\flat III/V$ is the backdoor dominant and $\flat V/V$ is the tritone substitution), but it also undoes the initial modulation of the branch, namely $key = \flat III$. The ensuing chord progression, unified by such reversals, also explains the apparition of original dominant and tonic chords in deeper levels of the syntactic derivation.

An analysis of the fundamental bass is relatively straightforward and reveals

[12] Clearly, tonicizations of local chords do not alter the perception of a circle of fifths.

similar results. Once we simplify the much fragmented first layer of modes (for example, considering the transient F major as a brief escape of the surrounding D minor), we obtain a similar form to that deduced from syntactic rules: $([iii] - I) - ([iii - vi] - I) - ([iii - ii] - I)$. By contrasting the different layers, we can reflect how the attempts to resolve $\mathrm{E}^{-7b5} - \mathrm{A}^7$ do not reach the tonic in a regular fashion (chained fifths) until the last of the three subsections. Unlike the previous approach (which prioritized the tension-generating structural gaps over conveying the expressiveness of the modulating change), we now possess the resources to describe both formal and harmonic phenomena without undermining the significance of any of the two.

A particular aspect of the application of structural modes is their capability to significantly portray robust harmonic sequences (such as circles of fifths) and to illustrate progressions with distant harmonies (like tritone substitutions) not necessarily as linear, locally contextualized elements, but rather as digressing segments which are linked as a whole to the main tonality. This is an essential feature to understand the filtering which occurs between layers, since $\mathrm{E}\flat_2^0$ and $\mathrm{F}_2^\flat$ can be thought of as temporary deviations from –respectively– $\mathrm{B}\flat_2^0$ and $\mathrm{D}_2^{\flat 3}$, the prevailing modal region in the particular fragments of the first and second section, respectively.

Nevertheless, the meaning of the transitions between the remaining modes remains to be discussed:

$$\begin{array}{ccccccccccccccc} \mathrm{D}_2^{\flat 3} & [\rightarrow\downarrow] & \mathrm{B}\flat_2^0 & || & \mathrm{D}_2^{\flat 3} & [\downarrow] & \mathrm{G}_2^{\flat 3} & [\rightarrow] & \mathrm{B}\flat_2^0 & || & \mathrm{D}_2^{\flat 3} & [\rightarrow\uparrow] & \mathrm{C}_2^{\flat 3} & [\rightarrow\uparrow] & \mathrm{B}\flat_2^0 \\ \sharp D & [\rightarrow\downarrow] & T & || & \sharp D & [\downarrow] & \sharp T & [\rightarrow] & T & || & \sharp D & [\rightarrow\uparrow] & S & [\rightarrow\uparrow] & T \end{array}$$

$[\rightarrow\downarrow]$ can be interpreted as the motion which leaves D minor suspended and ultimately "needs to be resolved". $[\rightarrow\uparrow]$ summarizes a double movement within a circle of fifths, so it progresses towards the main tonic as intended. Lastly, the $[\downarrow]$ between D and G minor can be seen as an approach to B♭ major, which is not completely fulfilled in a continuous, smooth manner, due to the $[\rightarrow]$ transition being applied: this leaves room for the third section to actually resolve the initial D minor appearance.

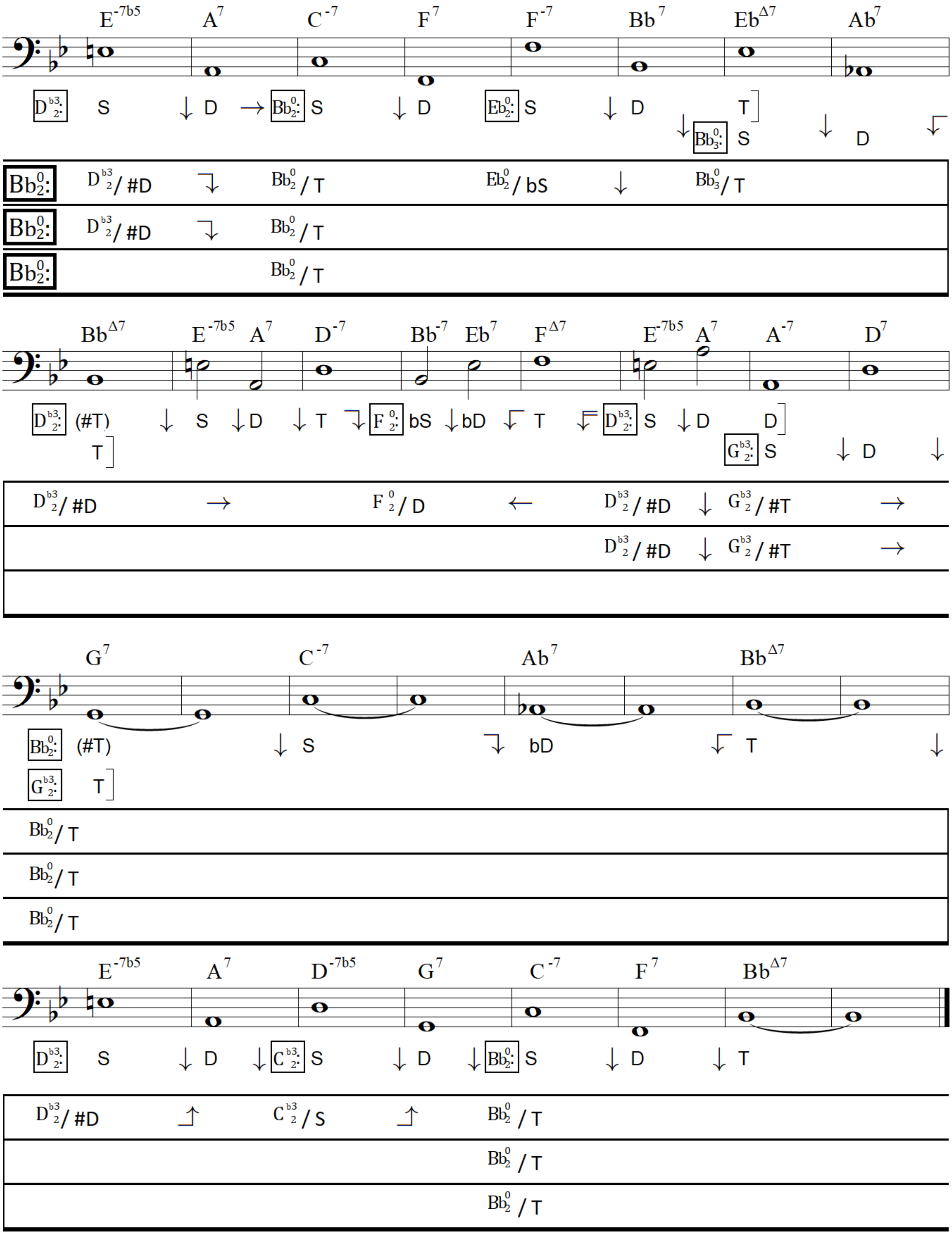

Figure 16: Analysis of *Stella by starlight* with recursive structural modes.

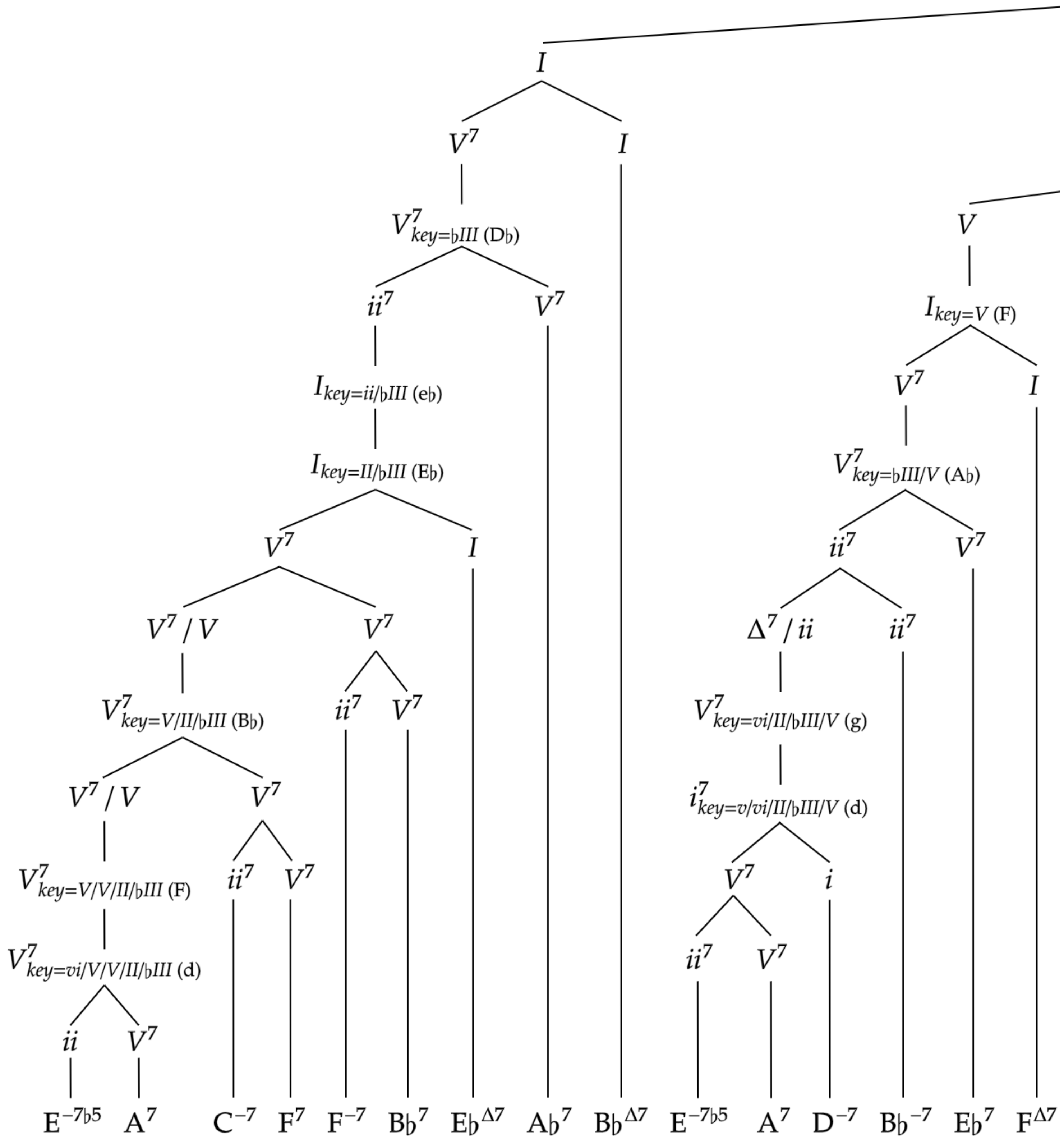

Figure 17: Syntactic analysis of *Stella by starlight* (left).

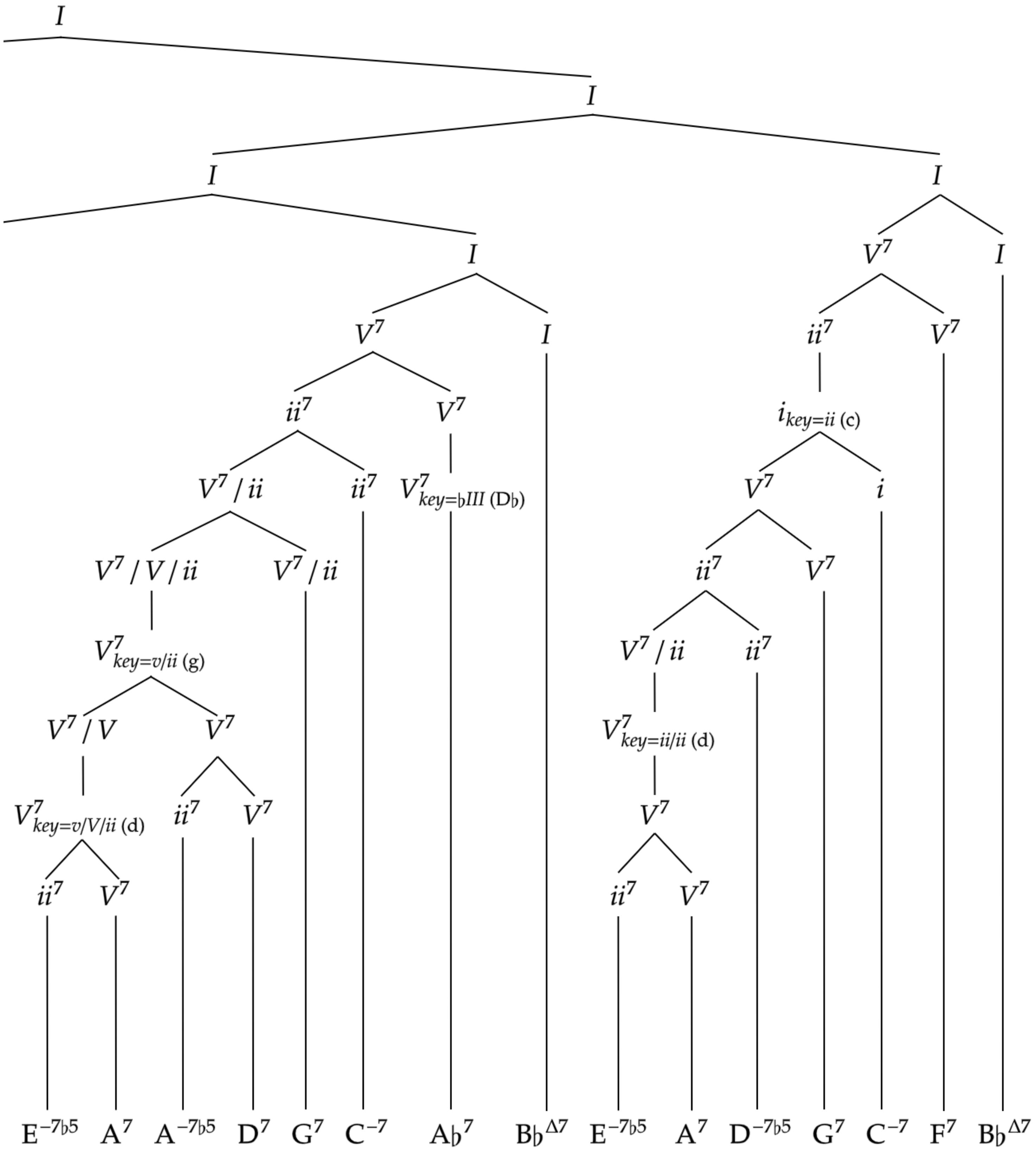

Figure 17: Syntactic analysis of *Stella by starlight* (right).

Nested structures: *My foolish heart*

In the first place, we will be studying the reharmonization popularized by musicians as Bill Evans. The original version, written by Victor Young, bases the entire first section on two turnarounds, even if the melodic contour (and the accompaniment in recordings such as Ethel Ennis' [Enn57]) parallel the climatic build-up of the new harmony. Nevertheless, it anticipates and helps us consolidate the role of a IV degree tonal region during the central section.

Taking a glance at the beginning of each melodic section, the analysis of structural modes shows a brief appearance of the chord E♭$^{\Delta 7}$ preceding a sudden leap towards a different tonality. Guessing the syntax of these elements can become a delicate task: the central section offers the possibility to resolve the chord either as a $\flat VI$ preparation of the local dominant or as a plagal derivation of the main tonic. This differentiates the passage from the occurrences of the "A" phrase (i.e. exposition and reexposition), as they can exclusively resort to the latter (since D^{-7} does not act as a dominant).

The peculiarity of *My foolish heart* resides in the tension-release structure which both harmony and melody are subscribed to: the defining local modes of the first 8 bars ascend diatonically from B♭ to D and descend afterwards. This musical intention is conveyed by both of our models. Grammatically, the left-branching structure reflects this "directed symmetry" by placing suspense-generating gaps where the modulations occur and resolving them afterwards along a unified branch. In terms of structural modes, the fundamental bass climbs and plunges along the circle of fifths (i.e. a sequence of structural progressions) following the transitions [←↓] and [→↑], which are equivalent to [↑↑] and [↓↓], respectively, and eventually balance out. The descent in the first section is achieved by splitting the local tonics of the modes into themselves and a second degree subdominant of the next modulation. Additionally, the sequence D^{7}−G^{-7} does not establish a G minor mode (since it is followed by a resolution to C^{-7}), but it foreshadows the modulation to the relative key which will take place during the last section.

Precisely, both systems cement the tonality of G minor in a segment of the last third of the tune, but –in the case of the syntactic approach– it comes at a cost. The key, as in a similar instance from *Stella by starlight*, is derived from the tonicization of the diatonic fifth of the second degree within B♭ major. However, this B♭ major arises from the reversal of a backdoor dominant through its relative minor key ($vi/\flat III$). The fact that no defining chord from B♭ major is reached and we still consider such a modulation to exist[13] responds to the commutative logic of substitutions, which settles a progression parallel to the expectable flow of fifths and is better represented by the malleability of fundamental bass notation.

Regarding the succession of the 8 ending chords, when it comes to the grammatical analysis, we opted for a continuous progression instead of dissociating the IV degree from the rest of the harmonic branch. This follows a similar reasoning to the reexposition of *Stella by starlight* in that both sequences carry a cadential momentum which often entails a preceding harmonic motion with some degree of directionality. Mike Melillo's and Chet Baker's recording [BM85] of the standard implicitly asserts such characteristics by replacing the progression by an analogous circle of fifths.

Delving into the middle section, the role of E♭$^{\Delta 7}$ still remains to be figured out in terms of its syntactical derivation. As it stands, each of the four modes which appear throughout measures 10-17 is considered to be originated from the subsequent key as a preparation. This conjecture is corresponded perceptually by the presence of a C in the melody which anticipates the seventh of D^7. Nevertheless, the rapid succession of the structural modes E♭, G minor, D minor and B♭ could imply a regular harmonic rhythm (also in terms of modulations) such that no tonality is particularly subjugated to another. This would motivate the possibility of understanding E♭ major as a long-term preparation of B♭ which could mimic the behavior of the IV degree in measures 1 and 17, and be correlated with the prevalence of IV as the defining structural mode in the middle section.

[13] Even if we simplify the subindex notation, as if no backdoor dominant had occurred.

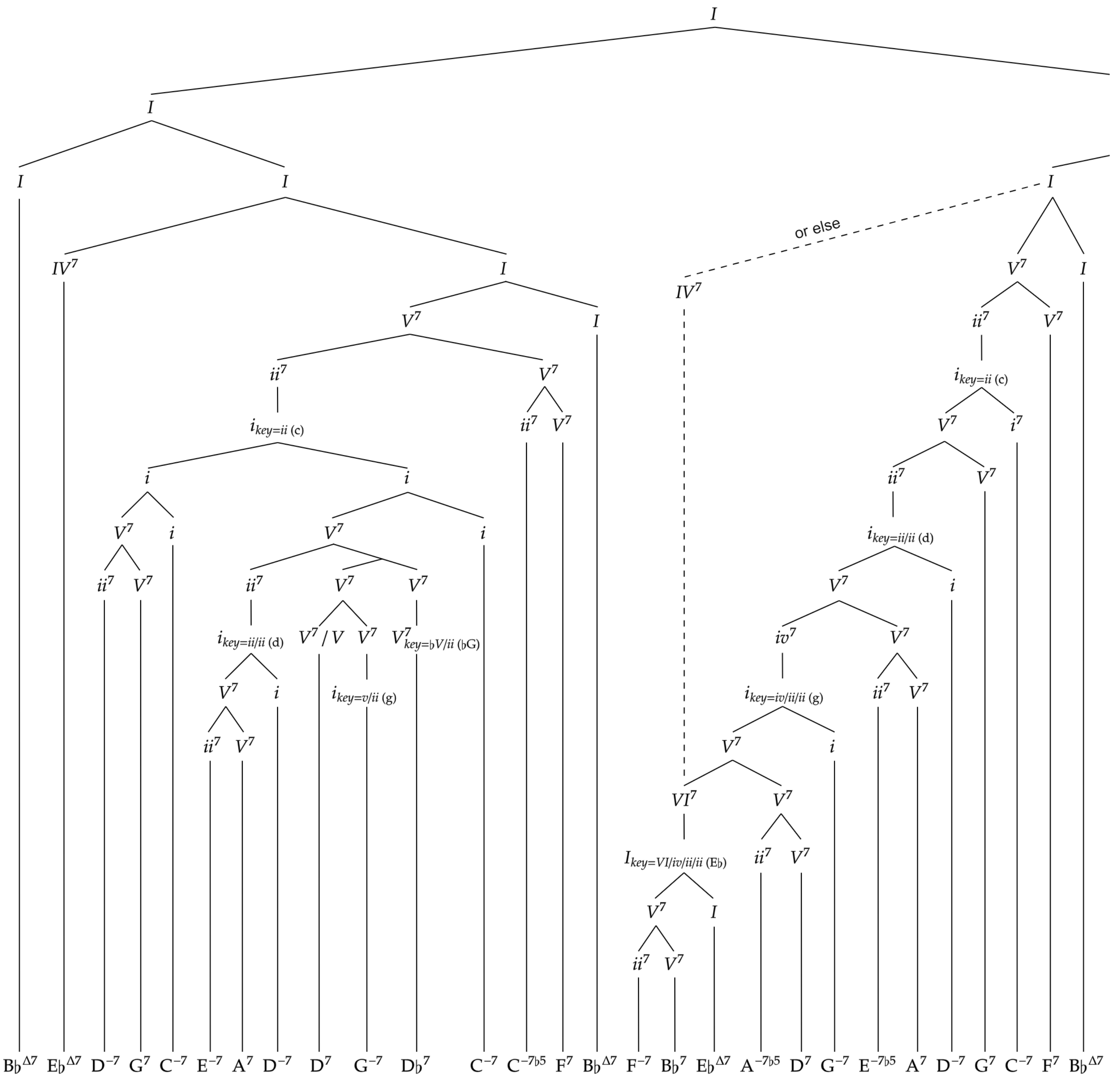

Figure 18: Syntactic analysis of *My foolish heart* (left).

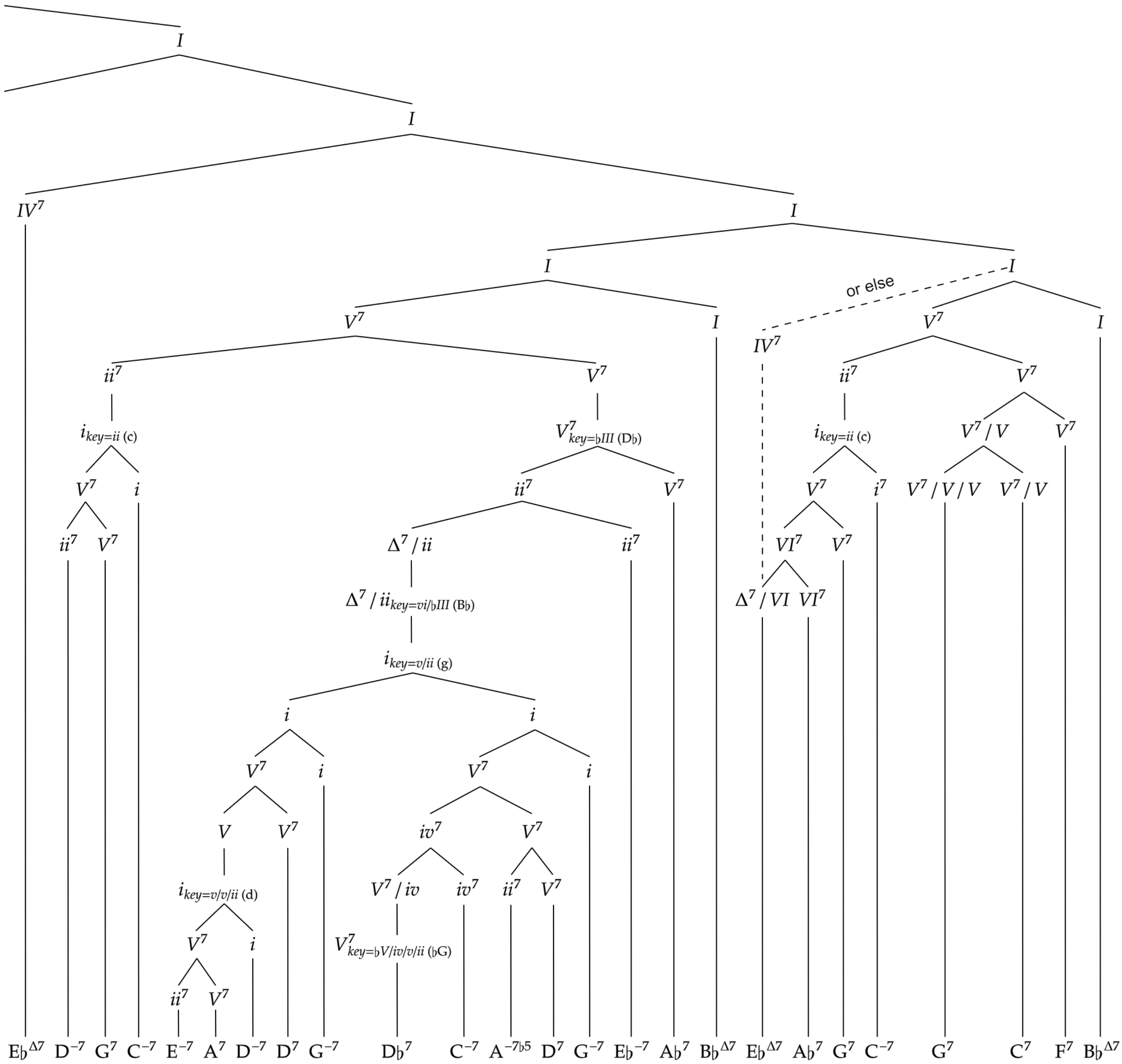

Figure 18: Syntactic analysis of *My foolish heart* (right).

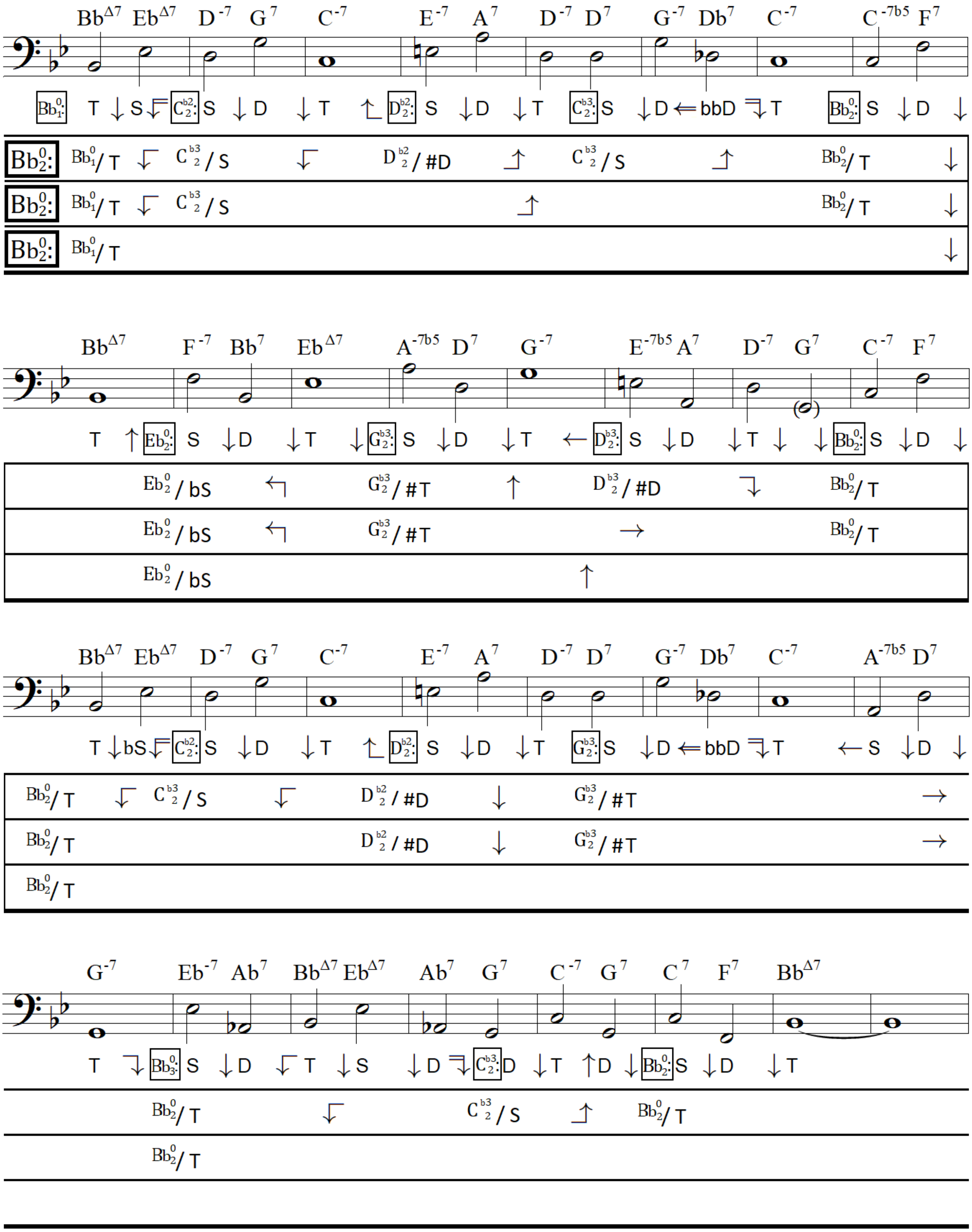

Figure 19: Analysis of *My foolish heart* with recursive structural modes.

Losing direction: *Giant steps*

When trying to employ any of the systems for the study of a tune like *Giant steps* or Bill Evans' *Orbit*, the principal hindrance appears to be the choice of a modal reference. Due to the cyclical nature of the harmony, one might even wonder if the resources are even applicable, in the sense that no significant insights might be derived from them. We will attempt to formalize an analysis of the standard and extract some conclusions.

There are two significant grammatical production rules used in this subsection, namely the preparation of the dominant $V \to \flat VI\ \ V$ and the derivation taking the inverse degree with respect to the tonal center, $I \to III\ \ I$, which acts as the opposite rule when paired with a tonicization of the III degree. We opted for a nested sequence of modulations to parse each of the two halves of the piece, rooted in a first degree of B major. Despite the fact that the changes are quite sudden on their own, arranging them in succession diminishes the generated tension (which we argue necessary to divide different modes into different branches), thus making the progression smoother and more uniform. However, if we were to connect the modes on the first half to their halfway parent $B^{\Delta 7}$, we would likely no longer need to invoke the additional rule $I \to III\ \ I$.

One major contrast between grammar-based and mode-based approaches, in this case, is that the changes can be effectively and explicitly described by production rules, but the analysis of the bass motion attributes a direct tonal function to each of the chords. From this perspective, the meaningful analytic freedom appears at the second layer of meta-modes: to derive it, we have chosen to systematically cancel out the contiguous inverse transitions (e.g. G[←↓]E♭[↑→]G becomes G), which roughly translates to interpreting any mode surrounded by two identical modes as a brief, unimportant modulation. Curiously enough, this results in a symmetrical global structure of the form B−G−E♭−G−B, a remarkably diverging outcome in comparison with the syntactical model.

Notice that no tonal functions have been assigned to the meta-modes, due to the fact that the modal center is not properly tonal in a broader scale. Despite almost edging out the tune from the parameters of tonality, the study of recursive structural modes provides a powerful tool to navigate the changes in a context of quasi-tonality. This stems from the fact that most of the arrow transitions are associated with a tradition of tonal chord changes and can usually be interpreted in a diverse amount of ways. For instance, $\leftarrow$ serves as a modulation to the relative minor key, but it can also be translated into $[\uparrow\uparrow\uparrow]$, which works as a structural alteration and as a link between other modes. Additionally, interpreting composite arrows as the juxtaposition of unidirectional arrows can prove useful by balancing out sharp-ward and flat-ward alterations (as seen in the tritone substitutions from *Round midnight*).

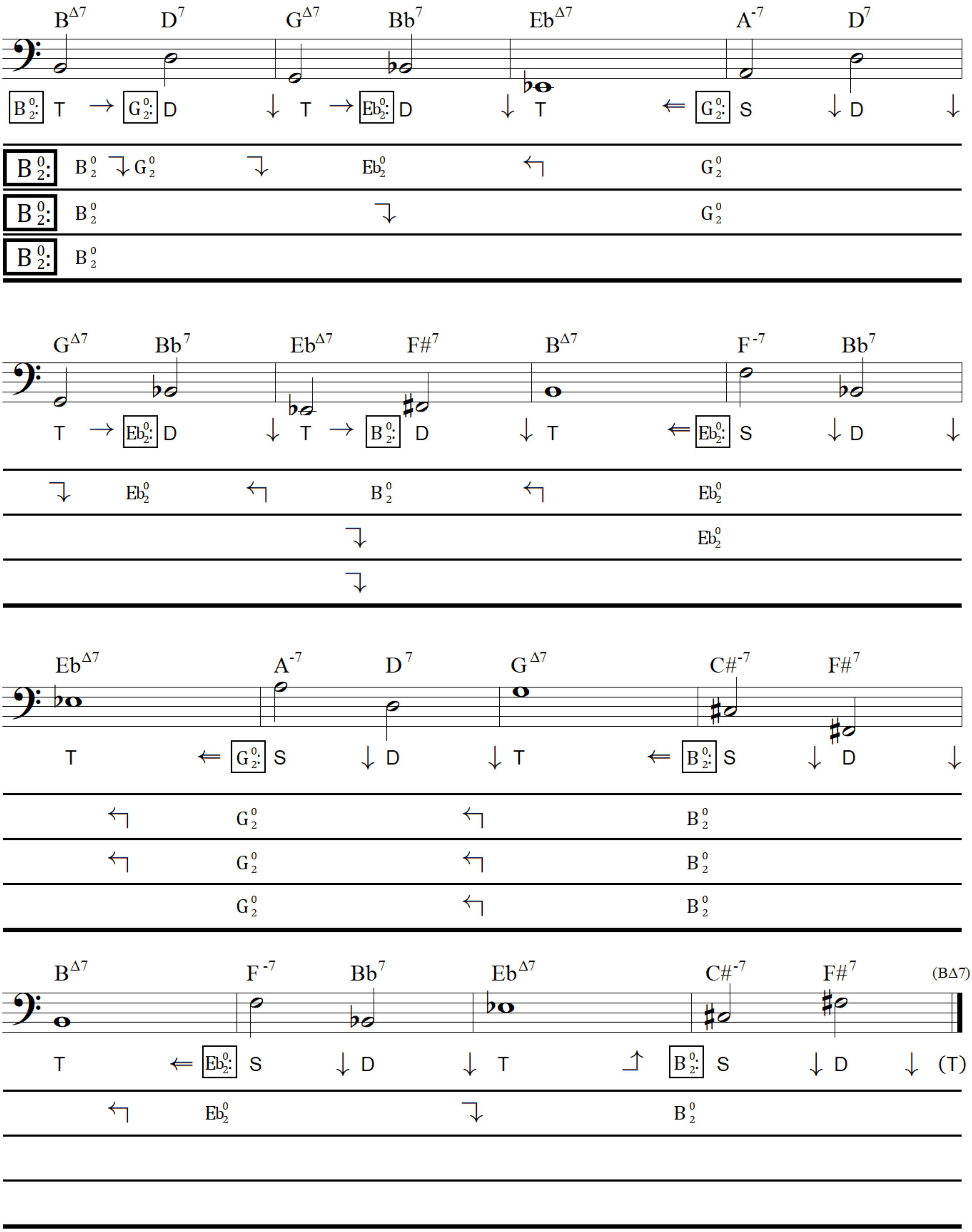

Figure 20: Analysis of *Giant steps* with recursive structural modes.

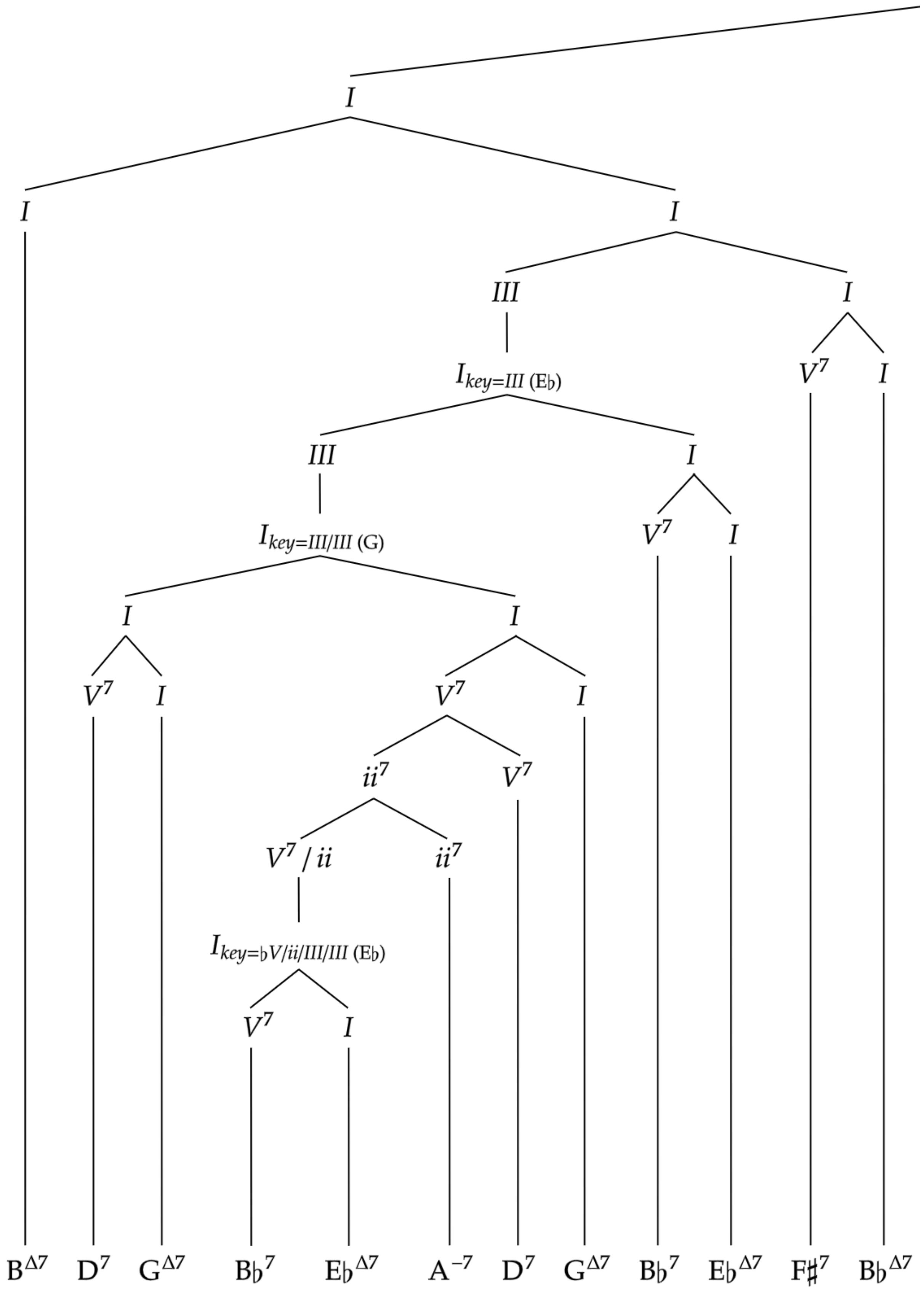

Figure 21: Syntactic analysis of *Giant steps* (left).

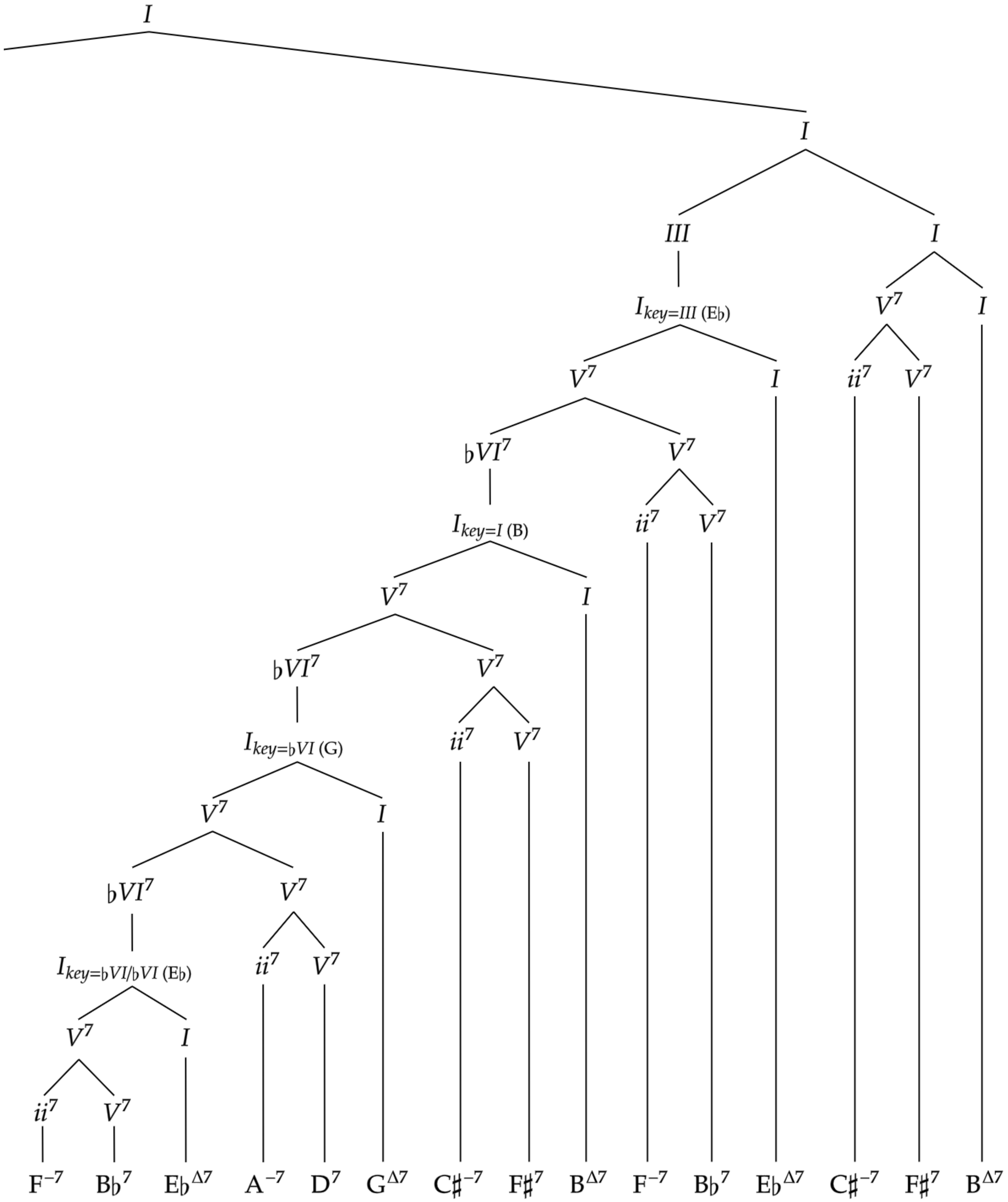

Figure 21: Syntactic analysis of *Giant steps* (right).

3.2 A hint at attribute grammars

We may now contemplate the reciprocal approach, that is, exploring how structural modes can have an influence, or even be integrated, in a harmonic syntax. After discussing some general aspects, we will study the interactions between grammars and automata with the goal of reaching a systematic method which we could then modify on the basis of fundamental bass analysis.

Taking into consideration the observations from the last section, one of the most evident additions for an extended syntactic model would be the production rules which balance out certain modulations. For instance, we resorted to backdoor dominants ($V/\flat III$) and tonicizations of the $\flat VI$, which are corresponded by their respective inverses vi and III. The former, as mentioned earlier, would complete the four possible substitutions within a semitone-tone scale over V^7, but –more importantly– it would also assimilate the axis of structural alterations ($\leftarrow$ and $\rightarrow$) introduced by the fundamental bass.

Another central feature of structural modes would correspond to the assignation of tonal regions: an improvement would come by allowing the key subindex notation to cancel out inverse modulations, so that a temporary departure to the tritone substitute would not require the previous chord to be accompanied by the subscript $key = \flat V/\flat V$. In order to envision a stricter interaction between models (by introducing tonal functions and formal modal regions), we would also need to classify the production rules in terms of how they condition the choice of tonality in a local scale. Some stabilize a tonality, such as $V \rightarrow IV\ \ V$, $X \rightarrow V/X\ \ X$ or the rules which involve leading tones and diminished chords; others usually imply a modulation, like substitutions on $\flat III$; and the remaining ones ($V \rightarrow \flat VI\ \ V$, diatonic fifth preparations, plagal rules, or even tritone substitutions) can either act as a development of the same tonality or as hinges.

In any case, we may also evade such a formalist transliteration and employ other resources which could work in favor of an integrated version of our current grammar. Recalling a result from section 1.4, we can assert the existence of a finite automaton which accepts exactly the words of a context-free grammar, if and only if such grammar is *regular*; that is, if its production rules are of the form $A \to Bx$ or $A \to x$,[14] where A, B are variables and x is a –possibly empty– word from the considered alphabet. Recall that degrees are represented by variables, and surface chords are the characters of the alphabet: this prevents the grammar from generating any sort of complex tree (beyond a main branch which only contains bifurcations in the shape of leaves) and, apparently, makes it impossible to recreate the harmonic sequences of Rohrmeier's grammar. The difficulty lies in the fact that we can not have two variables at any state of the production process, since the rules "substitute" the current one.

However, if we set out to construct a regular grammar from scratch, we can propose to model the standards of the form ABC (most likely ABA') where each section is generated in one branch (in Rohrmeier's sense) by introducing a grammar according to the following criteria:

- We will consider a subset of the rules presented in the previous sections.
- Each variable must represent a degree (including $\flat X$ and $\sharp X$), and there must exist rules which substitute the variable by its preparation juxtaposed to their actual surface chord (for example, $I \to V\ \mathrm{C}^{\Delta 7}$).
- In order to compensate the lack of proper branching, we will generate copies of the variables and the rules for each of the three sections of the standard, and enumerate them with subindices 1, 2 and 3, respectively. Then, we will add two collections of rules to allow a free switch on the borders of the sections (that is, the first chord of a section can be preceded by any degree):
- Let $ch(X)$ symbolize the surface chord of a degree X. Consider any section $n \in \{2,3\}$, its possible first degrees (Y_n) and the last from the previous section

[14] At the beginning of the text, we presented regular grammars by introducing the left-branching rule $A \to xB$. Both formulations (called, respectively *right-linear* and *left-linear* induce equivalent regular grammars, as long as a single type is used.

(X_{n-1}). If we include the production rules $Y_n \rightarrow \tau_n \;\; ch(Y_n)$ and $\tau_n \rightarrow X_{n-1}$, we will be capable of transitioning between sections over the chords $X_{n-1} - Y_n$.

We will exemplify the process of generating a sequence of chords from this kind of syntax by considering a simplified variation of the harmony in *Autumn leaves* (as presented in the lead sheet during the appendix). The grammar will consist of the following elements: a set of variables $Var = \{i_n, ii_n, III_n, \flat IV_n, iv_n, V_n, VI_n, \flat VII_n, VII_n, \flat I_n \mid 1 \leq n \leq 3\}$, an alphabet $\Sigma = \{\text{A}^{-7}, \text{B}^{-7\flat5}, \text{C}^{7}, \text{D}^{-7}, \text{E}^{7}, \text{F}^{7}, \flat\text{G}^{7}, \text{G}^{7}, \flat\text{A}^{7}\}$, a starting variable i_3 and the following set P of production rules:[15]

1. Prolongation: $X_n \rightarrow X_n \;\; ch(X_n)$ | 2. Dominant preparation: $X_n \rightarrow V/X_n \;\; ch(X_n)$
3. Diatonic fifth prep.: $X_n \rightarrow \Delta/X_n \;\; ch(X_n)$ | 4. Tritone substitution: $X_n \rightarrow \flat V/X_n$
5. Section link (R): $X_n \rightarrow \tau_n \;\; ch(X_n)$, $n > 1$ | 6. Section link (L): $\tau_n \rightarrow X_{n-1}$, $n > 1$

Notice that, since we consider tritone substitutions to be a plain change of variables, we need to complement them with other rules for them to have a repercussion in the resulting chord progression. The following expressions summarize the generation process of the chord sequence in *Autumn leaves* – it also begins from the right hand side, and indices under bold arrows denote the rule which is being applied:

<u>Full changes:</u> (A section) $\text{D}^7 \; \text{G}^7 \; \text{C}^{\Delta7} \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5} \; \text{E}^7 \; \text{A}^{-7}$ |

(B sect.) $\text{B}^{-7\flat5} \; \text{E}^7 \; \text{A}^{-7}\text{D}^7 \; \text{G}^7 \; \text{C}^{\Delta7} \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5} \; \text{E}^7 \; \text{A}^{-7} \; \text{A}\flat^7 \; \text{G}^{-7} \; \text{G}\flat^7 \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5} \; \text{E}^7 \; \text{A}^{-7}$ |

(A′ section) $\text{D}^7 \; \text{G}^7 \; \text{C}^{\Delta7} \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5} \; \text{E}^7 \; \text{A}^{-7}$

<u>Derivation of A′:</u> $i_3 \Rightarrow_2 V_3 \; \text{A}^{-7} \Rightarrow_3 ii_3 \; \text{E}^7 \; \text{A}^{-7} \Rightarrow_3 \ldots \Rightarrow_3 iv_3 \; \text{G}^7 \; \text{C}^{\Delta7} \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5} \; \text{E}^7 \; \text{A}^{-7}$

<u>Link between B and A′:</u> $iv_3[\ldots] \Rightarrow_5 \tau_3 \; \text{D}^{-7}[\ldots] \Rightarrow_5 i_2 \; \text{D}^{-7}[\ldots]$

<u>Tritone substitutions:</u> $VI_2 \; \text{B}^{-7\flat5}[\ldots] \Rightarrow_3 III_2 \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5}[\ldots] \Rightarrow_4 \flat VII_2 \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5}[\ldots] \Rightarrow_2$ $\flat IV_2 \; \text{G}\flat^7 \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5}[\ldots] \Rightarrow_4 VII_2 \; \text{G}\flat^7 \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5}[\ldots] \Rightarrow_3 IV_2 \; \text{G}^{-7} \; \text{G}\flat^7 \; \text{F}^{\Delta7} \; \text{B}^{-7\flat5}[\ldots]$

<u>Link between A and B:</u> $ii_2[\ldots] \Rightarrow_5 \tau_2 \; \text{B}^{-7\flat5}[\ldots] \Rightarrow_5 i_1 \; \text{B}^{-7\flat5}[\ldots]$

[15]Recall that each of the entries represents a set of rules according to the values which X_n can adopt.

One could discuss whether the application of the section rules was required during the return to A', if they understood that the reexposition is the resolution and the continuation of the resolution started at the beginning of the B theme. At any rate, we can now design a finite automaton which will identify the same chord sequences as the constructed regular grammar. It will consist of a set of states $K = Var \cup \{f\}$ (where f is a new symbol which acts as the only accept state), our alphabet Σ, a start state $q_0 = i_3$, and a set of transitions

$$\Delta = \{(A, x, B) \mid (A \to Bx) \in P\} \cup \{(A, x, f) \mid (A \to x) \in P\}.$$

The automaton reads a harmonic progression from right to left and moves between states (degrees) by applying the rule/transition which corresponds to the next chord. The computation ends if there are no possible transitions at the current state given a chord, or if the accept state is reached (i.e. a transition which is analogous to the rule which replaces the variable with just a surface chord). Figure 22 us illustrate it by considering only a selection of states and transitions related to the A section (which excluded tritone substitution and the latter half of the section links):

The automaton begins its path on the start state (τ_2), and immediately travels to i_1 following the transition triggered by A^{-7}. After that, the next state V_1 is reached by the application of a dominant preparation. The rest of the chords are derived from the movement in diatonic fifths, navigating the automaton clockwise until the initial iv_1 chord is reached. Lastly, since no chords remain, we can apply the transition associated to the empty word λ to reach the accept state.

The fact that our new grammar demands an instantaneous replacement of each new variable and prevents us from stacking them, produces an impractical amount of states. If we wanted to at least simplify some of the ensuing production rules by relying on the prolongation rule (i.e. $i_1 \to v_1\ \text{A}^{-7}$ would be handled by two simpler transitions $i_1 \to i_1\ \text{A}^{-7}$ and $i_1 \to v_1$), we would be allow to apply consecutive instances of fifth-related rules ($i_1 \to v_1$, $v_1 \to ii_1$, $vi_1 \to vi_1$, etc.). Thus, we would

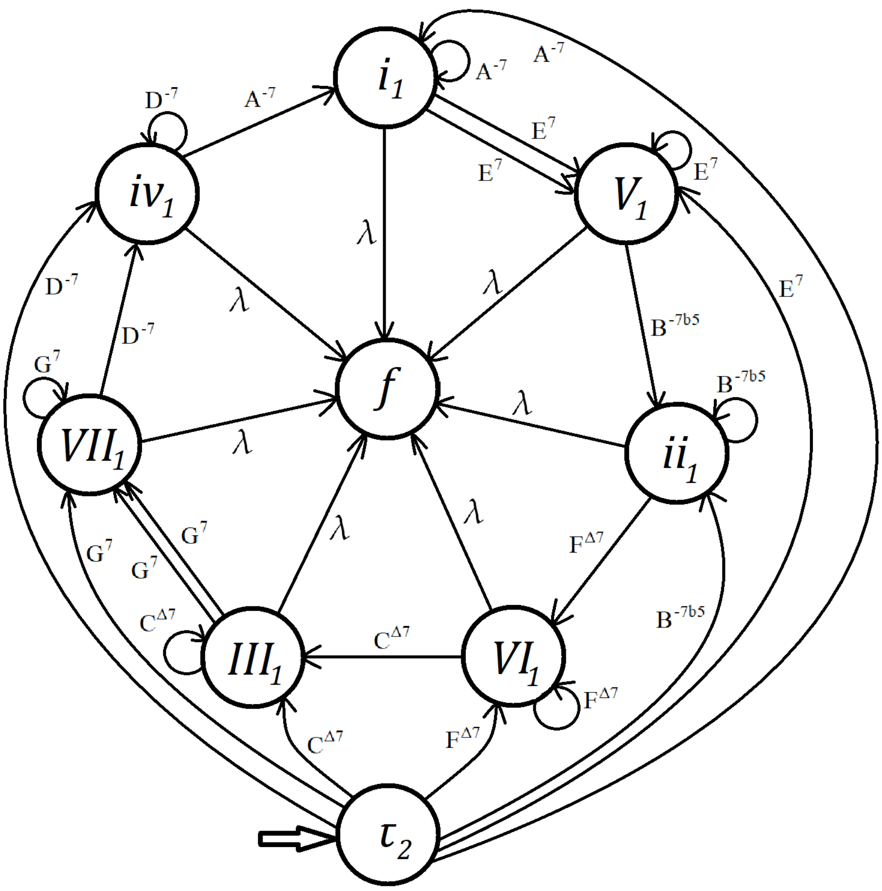

Figure 22: Automaton which, among other sequences, accepts the initial 8 bars of *My autumn leaves*.

effectively be able to generate any succession of diatonic chords with no internal logic whatsoever.

Even if the complete automaton is suited to identify other successions, it becomes far too cumbersome to consider any larger structures which could better be addressed by other grammars with a higher expressive potential. Adding more rules and phrase frameworks would make our approach more comprehensive, but not as accurate and definitely less optimal than Markov-based models, which defeats the purpose of opting for a formal generative grammar. Furthermore, we would still have to tackle issues like modulating regions, thus entailing more complexity.

Thorough adaptations of the syntax

There is still version of the equivalence theorem for finite automata which encompasses the entirety of context-free grammars (including ours) – the existence of a context-free grammar which can generate a language L is contingent on the existence of a *pushdown automaton* which accepts L, and vice versa. As mentioned in the introductory chapter (and shown in [LP98]), this kind of machine consists of the same elements as nondeterministic automata (a set of states K, an alphabet Σ, a start state q_0, a set F of accept states, and a collection Δ of transitions), but it also includes a supplementary alphabet Γ for the stack, a modifiable string of characters. In addition, the transitions are adapted to the new functioning, so they take the form $((p,a,b),(q,x))$, where $p,q \in K$, $a \in \Sigma \cup \{\lambda\}$, $b \in \Gamma \cup \{\lambda\}$ and $x \in \Gamma^*$.

Each computation step unfolds according to its transition $((p,a,b),(q,x))$ as follows: the machine moves from state p to q, as it reads a –possibly empty (λ)– character a from the input string; then, it replaces b in the stack with the word x (which can result in a simple concatenation or a removal, since b and x can again be empty). We represent the condition of a pushdown automaton in a certain computational stage (called a *configuration*) as $\alpha p x$, where α is the current content of the stack, p is the state and x is the unread portion of the input word. Then, we say that the machine *accepts* a word x if it can reach the configuration $\lambda q \lambda$ from an initial $\lambda q_0 x$, where $q \in F$. That is, it recognizes x if it can reach an accept state by starting and ending the process with an empty stack, regardless of its actual content throughout the process.

The prominence of the aforementioned equivalence theorem resides in its proof ([Mar21]), an algorithm to deduce the sought automaton from any context-free grammar $G = (V,\Sigma,P,S)$. It will be expressed as $M = (K,\Sigma,\Gamma,\Delta,q_0,\{f\})$, where $K = \{q_0, f\}$, $\Gamma = V \cup \Sigma$ and Δ is made up of the transitions:

1) A starting $((q_0,\lambda,\lambda),(f,S))$, which switches to the accept state and adds the starting variable to the stack.

2) For every rule $A \to x$ in P ($x \in (V \cup \Sigma)^*$), a transition $((f, \lambda, A), (f, x))$, which recreates the production rule within the stack by substituting A with x.

3) For every character $a \in \Sigma$, a transition $((f, a, a), (f, \lambda))$, which erases a from the stack if the symbol appears as an input.

To visualize how a word is recognized by M, consider that the machine will add the initial variable S to the stack in the first place – then, a type 2 transition will replace it with the right hand side of a grammatical rule which transforms S into something else. Then, it can gradually reconstruct the input word over the stack by deriving the leftmost element until it is a character from Σ. If the automaton is equivalent to the grammar, such a character can actually match the current symbol of the input stream, in which case a type 3 transition can be applied to remove the the element from both strings. Gradually, we are able to recreate (bit by bit) the word in the stack and erase it as the input coincides.

This provides an extremely powerful method to operate syntactic models of harmony. We can showcase it by analyzing the sequence $\mathrm{G}\flat^{7}-\mathrm{F}^{\Delta 7}-\mathrm{B}^{-7\flat 5}-\mathrm{E}^{7}-\mathrm{A}^{-7}$ from the final 5 measures in *Autumn leaves*' B section. As always, our basic alphabet consists of surface level chords, but the one for the stack also includes all scale degrees (even their altered versions, with i being the start state as well). The subset of transitions which we will consider corresponds to the enforced rules:[16]

1) Dominant or diatonic fifth preparations: $((f, \lambda, X), (f, \Delta/X\ \ X))$.

2) Tritone substitution: $((f, \lambda, III), (f, \flat VII))$.

3) Translation of the degrees into surface chords: $((f, \lambda, X), (f, ch(X)))$.

4) The initial $((q_0, \lambda, \lambda), (f, i))$.

5) For every chord $a \in \Sigma$, $((f, a, a), (f, \lambda))$.

[16]Mind the reordering of the list indices.

Then, our automaton reads the chord sequence as follows:[17]

$$[\lambda]q_0[\text{G}\flat^7\ \text{F}^{\Delta 7}\ \text{B}^{-7\flat 5}\ \text{E}^7\ \text{A}^{-7}] \Rightarrow_4 [i]f[\text{G}\flat^7\ \text{F}^{\Delta 7}\ \text{B}^{-7\flat 5}\ \text{E}^7\ \text{A}^{-7}] \Rightarrow_1$$
$$[V\ \ i]f[\text{G}\flat^7\ \text{F}^{\Delta 7}\ \text{B}^{-7\flat 5}\ \text{E}^7\ \text{A}^{-7}] \Rightarrow_1 \ldots \Rightarrow_1 [III\ \ VI\ \ ii\ \ V\ \ i]f[\text{G}\flat^7\ \text{F}^{\Delta 7}\ \text{B}^{-7\flat 5}\ \text{E}^7\ \text{A}^{-7}] \Rightarrow_2$$
$$[\flat VII\ \ VI\ \ ii\ \ V\ \ i]f[\text{G}\flat^7\ \text{F}^{\Delta 7}\ \text{B}^{-7\flat 5}\ \text{E}^7\ \text{A}^{-7}] \Rightarrow_3 [\text{G}\flat^7\ \ VI\ \ ii\ \ V\ \ i]f[\text{G}\flat^7\ \text{F}^{\Delta 7}\ \text{B}^{-7\flat 5}\ \text{E}^7\ \text{A}^{-7}]$$
$$\Rightarrow_5 [VI\ \ ii\ \ V\ \ i]f[\text{F}^{\Delta 7}\ \text{B}^{-7\flat 5}\ \text{E}^7\ \text{A}^{-7}] \Rightarrow_{3,5} \ldots \Rightarrow_{3,5} [i]f[\text{A}^{-7}] \Rightarrow_3 [\text{A}^{-7}]f[\text{A}^{-7}] \Rightarrow_5 [\lambda]f[\lambda]$$

This way, it is possible to make the pushdown automaton recognize each of the standards we have studied in section 3.2 after including the transitions which correspond to the remaining syntactical rules.

Having examined varied ways to handle grammars with an extra layer of formalization, we can take a step further and involve aspects of structural modes as additional elements of our syntaxes. For that purpose, following [Knu68], we define a (finite) *attribute grammar* to be a pair (G, ι), where G is a context-free grammar (deterministic or not, as we defined them earlier) and ι is a function which maps words of G to a *semantic* interpretation.

The essence to this kind of constructions lies in the attribution of values, which makes use of production rules to infer the interpretations of other elements according to those we already know. For example, a grammar G_+ with alphabet $\Sigma = \mathbb{N} \cup \{+\}$ and production rules Term $\rightarrow$ Term $+$ Term and Term $\rightarrow n$ (for every integer n) can be used to reproduce natural summation: a final word such as $10 + 2$ can be generated by G_+ as $S \Rightarrow S + S \Rightarrow 10 + S \Rightarrow 10 + 2$, but this is just a syntactic construction without our usual interpretation of the sum. If we define ι to return either the summation of descendants (for the variables which unfold into the expression of a sum) or a numeric value (if they produce a natural number), the interpretation of $10 + 2$ is $\iota(10 + 2) = \iota(10) \dot{+} \iota(2) = 10 \dot{+} \iota(2) = 10 \dot{+} 2 = 12$, the expected value of the sum.[18]

[17] The subindices of arrows actually denote one of the rules among the specified class. We use brackets to distinguish the stack, the current state and the processed word.

[18] $\dot{+}$ represents the proper arithmetic operator, which we define to act on the symbols of $\Sigma \setminus \{\cup\} = \mathbb{N}$.

The attributive function of G_+ is an instance of a synthetic approach, which derives the values of ancestors based on the interpretations of the expressions they generate. In other words, it is a bottom-up process which resembles the establishment of higher tonal regions in the analysis of recursive structural modes. Per contra, in order to assign tonal functions to the elements of our grammar, it is likely more effective to employ inherited attributes, which evaluate the initial variable and pass down the corresponding interpretations to the remaining stemming elements. In particular, we can adopt R-attributed grammars as the most suitable and well-spread top-down system for our syntax: given a production rule $A \to X_1 \ X_2 \ \dots \ X_n$, the value $\iota(X_i)$ is surmised from the interpretations $X_1, \dots, X_{i-1}$ and A.[19]

Now, given the formalization of Rohrmeier's syntax for jazz standards, $G = (Var, \Sigma, P, i)$, we can establish an R-attributed grammar to associate degrees and chords with tonal functions from the structural approach. Recall that variables in Var represent the scale degrees, Σ the surface chords, and P are the production rules. Let us inspect the latter and ascribe an interpreting function ι.

In general, we opt for second structural modes, since they match the grammatical parsing of $ii - V - i$ as $S - D - T$. In the case of minor keys, we will also attempt to define ι so that it regards $iv - VII - III$ as $\flat S - \flat D - \flat T$, making the relative tonality possess a parallel collection of functions. The remaining of the diatonic degrees, VI, can be considered a $\sharp T$ if it is connected to S, and the altered degrees will receive their function in accordance to their corresponding modulation. Therefore, we have defined a set $I = \{\sharp T, S, D, T, \flat S, \flat D, \flat T\}$ of tonal functions which acts as a counterpart of the degrees and becomes their image under ι. We also translate the arrow notation in structural modes as a set of functions (e.g. $\downarrow(\iota(V)) = \downarrow(D) = T$) and add a clause to ensure that the cyclical motion along the circle of fifths is consistent, that is, $\downarrow(\flat T) = \downarrow(\iota(III)) = \iota(VI) = \sharp T$. The following table relates the production rules of the grammar to interpretation which ι makes of the newly generated element

[19] In the case of non-strictly inheriting grammars (which also involve synthesis), only the inherited attributes must be considered for A.

$g(X)$ (i.e. $X \to g(X)\ \ X$ for preparation and prolongation rules, and $X \to g(X)$ for substitutions):

Rule	Expression	Value of $\iota(g(X))$
Diatonic fifth	$X \to \Delta/X\ \ X$	$\uparrow(\iota(X))$
2^{ry} dominant	$X \to V/X\ \ X$	$\uparrow(\iota(X))$
Leading tone	$X \to vii^{\circ}/X\ \ X$	$\uparrow(\iota(X))$
Half cadence 1	$V \to \flat VI\ \ V$	$\sharp T$
Half cadence 2	$V \to IV\ \ V$	$\flat S$
Diminished 1	$X \to X^{\circ}\ \ X$	$\downarrow(\iota(X))$
Diminished 2	$X \to \flat ii^{\circ}/X\ \ X$	$\leftarrow(\iota(X))$
Half-diminished	$X \to ii^{7\flat 5}/X\ \ X$	$\downarrow(\iota(X))$
Plagal cadence	$I \to IV\ \ I$	$\downarrow(\iota(X))$
Substitutions	$X \to Sub/X$	$\iota(X)$
Backdoor V	$V/X_{key=Y} \to V_{key=\flat V/X/Y}$	D
Tritone subs.	$V/X_{key=Y} \to V_{key=\flat III/X/Y}$	D
Tonicization	$X_{key=Y} \to I_{key=X/Y}$	T
Prolongation	$X \to X\ \ X$	$\iota(X)$

Table 3.1: Interpretation of the generated variables depending on the rule applied.

A few remarks: we assume that substitutions do not alter the character of a chord since they act as a temporary replacement which does not modulate (otherwise, it would acquire a functionality within the new tonality); also, when modulating and involving the subindex key notation, we adhere to the formal rule, since any additional tweak would change the role of the current degree. On a related note, instead of possibly analyzing backdoor dominants as dominants of a third mode, we only consider the modulating syntactic rule. Lastly, notice that the prospect of non-structural progressions is ruled out, because they emerged as a solution to an initial flaw of structural modes which does not occur in this grammar (since a ramification or a modulation could portray the progression in question). As an example, the tonal functions in the A section of *Autumn leaves* are deduced by the recursive application of diatonic fifth rules, and the tonicization of III: $S - D - T - \sharp T - S - D - T$.

Chapter 4

Proposing a mixed model

The previous chapter introduced attribute grammars as a theoretical construction to include information about modes within a syntactic method of analysis. However, trying to add more data or even attempting to refine the formalization will surpass the capabilities of an attribute-based approach. Without resorting to brute force or statistical quantifications of the likeness between models, we can suggest a new construction which conveys information from both systems of analysis and has a direct implementation.

In this chapter, we will present Turing machines as a more powerful tool to demarcate languages and describe and run algorithms – in fact, there is no known mathematical object ([Cop20]) which outperforms them in this aspect, even though there exist improvements which do reduce the time computation. With these, we will be able to present a hybridized algorithm to parse the harmonic progressions which concern this work.

4.1 Considerations about the suggested model

Let us begin by narrowing down the features of the previous systems which we want to include. Ideally, the entirety of both of them can be preserved at the cost of a very sophisticated algorithm, but we ultimately need to forfeit some precision in order to obtain a more solid formalization which is neither completely rigid nor general enough to generate any chord sequence.

We will consider Rohrmeier's assumption of a right-headed, left-branching harmony, by incorporating a series of transitions analogous to a reduced subset of his production rules. On the other hand, we will strengthen the interaction of the remaining ones with the objective of following an intuitive deterministic course of action (which lends itself to an immediate programming perspective). We will achieve this by presuming that harmonic motion occurs foremost in accordance to an implicit presence of the circle of fifths, among others.

Up to some extent, this is a shared feature with structural modes. However, the attribution of tonal functions and modal regions is a distinct feature that we will still be including. We do not consider the notation for the fundamental bass superfluous (even having considered such an amount of information), but its addition in the new model would not compensate the supplementary resources we should use.

Our goal is to implement a system which can follow the generation process from right to left, by gradually adding the branches and their ramifications as they appear. For each new chord, the automaton should reach a state which represents its corresponding degree, while it eventually writes the mode it belongs to and the tonal function it develops within.

An enhancement which was finally discarded from the definitive version (for the sake of efficiency) consisted in integrating the depth level of branches and modulations as well. That is, the resulting product of the model would have shown how many bifurcating production rules and keys every particular chord would have needed to go through. This would have established a direct, graphical correspondence between the result and the outline of their related syntactic trees.

We will now introduce the concept of *Turing machine*: it works as an upgraded version of finite automata, in which the tape can move in either directions and can be rewritten or expanded infinitely on one end. The tape is now semi-infinite, since it begins with an initial marker $ (which can not be trespassed), it contains the input

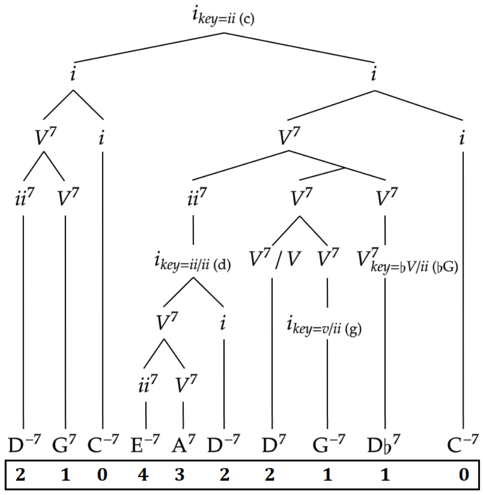

Figure 23: Segment of the grammatical analysis of *My foolish heart*, showing the relative chordal depth. Note that preparation rules only increase the value of the new degree.

word and is filled with infinite blank characters ($*$). In contrast with other machines, conceive a *cursor* which points to the current position of the tape, so that we can later consider more complex variations. As described by [LP98], the elements of a Turing machine $M = (K, \Sigma, \Gamma, \delta, q_0, q_F)$ are:

1) A finite set of states K.

2) An alphabet Σ for the input word, which does not contain \$ or $*$.

3) An alphabet Γ for the tape, which includes Σ, \$ and $*$.

4) A start state $q_0 \in K$.

5) A single accept state $q_F \in K$.

6) A (partial) function $\delta : (K \setminus \{q_F\}) \times \Gamma \to K \times \Gamma \times \{L, R, S\}$ of transitions, where L (left), R (right) and S (still) denote the movement of the cursor. Additionally, δ is forbidden to write \$ anywhere but at the beginning, and whenever \$ appears, the cursor can not move left.

Starting at the state q_0, the automaton can apply the transition $\delta(q,a) = (p,b,i)$ to perform the following tasks in a single step: transition from the state q to p, erase the character a from the tape (if the cursor is pointing it) and add b, and move the cursor one cell in the direction i. M is said to accept a word x if the process derived from x eventually stops and it does so at the state q_F. Nevertheless, Turing machines are highly regarded because the resulting tape can convey information or "execute the instructions" of an algorithm.

Let us illustrate this with an example from [Mar21]. M is defined from $K = \{q_0, q_F\}$, $\Sigma = \{0,1\} \subseteq \{0,1,*,\$\} = \Gamma$, and the transitions

$$\delta(q_0,0) = (q_0,1,R), \quad \delta(q_0,1) = (q_0,0,R), \quad \delta(q_0,*) = (q_F,*,S).$$

In other words, the machine swaps the appearances of 0 and 1 in the tape, and moves rightwards until the entry word is exhausted. Since it transitions from q_0 to q_F during the last step, any word which consists of zeros and ones is accepted, even the empty word. As in the case of simple finite automata, we can depict M as a graph by writing (a,b,i) as a summary of the transition $\delta(q,a) = (p,b,i)$:

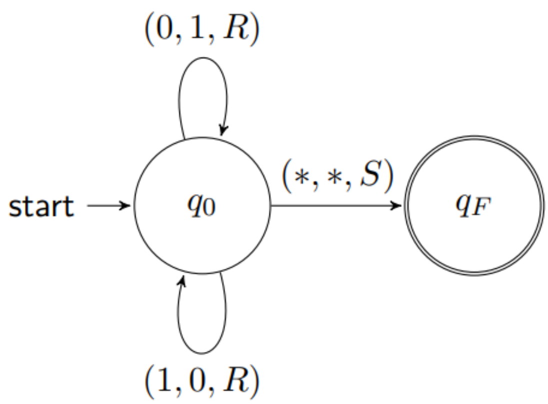

Figure 24: A Turing machine which exchanges the occurrences of 0 and 1 in a word, as presented in [Mar21].

4.2 Formalization and implementation

We will conclude the work by applying the presented formalization of Turing machines to the analysis of harmonic progressions. The current model works as a prototype and it can be easily expanded to include every rule in the previously studied systems by considering a non-deterministic machine. We opted for a deterministic one due to its versatility, and it includes transitions for the prolongation, dominant and diatonic fifth preparation, plagal cadence, modulation and tritone substitution. Not only will the model accept the sequence we will propose, but it will also provide information about its modes, tonal functions and –indirectly– inner structure.

On account of an equivalence theorem ([LP98]), we can consider Turing machines with several tapes knowing that they could be converted into a single tape for another purposes. They operate in a very similar fashion, with the exception that the remaining tapes only contain the starting symbol other than empty characters, and the cursor can move independently over each of them. This allows for more efficient and conceptually intelligible algorithms. However, our cursor will mostly progress uniformly and the tape which contains the input word will only be altered through the duplication of some chords.[20]

Let us properly introduce our Turing machine $M = (K, \Sigma, \Gamma, \delta, q_0, q_F)$. The *alphabet* of the input, Σ, is reserved for the actual chords, and along with the names of the 24 major and minor modes, the set of tonal functions $\{\sharp T, S, D, T, \flat S, \flat D, \flat T\}$, and the symbols $\{\$, *\}$, it constitutes Γ. The first tape will represent the chord succession, while the second and third ones will determine –respectively– the current key and tonal function, and the last one will act as an immutable auxiliary copy of the first (to compare which chords appear at a more basic generation stage).

Regarding the *states*, an initial q_0 is held until the cursor reaches the last chord, with the purpose that the automaton can go through the sequence backwards. There

[20] Accordingly, we will be refering to the *main* and *auxiliary* positions of the cursor.

are separate collections of states corresponding to the sets of degrees of each tonality. Hence, every modulation (even tritone substitutions) redirects the flow to an independent set. Besides from the mandatory accept state, there exists an *error state* which is reached whenever the subsequent chord is not derivable from our axiomatic. As the name suggests, it can not be abandoned and prevents the machine from accepting the word in question.

In order to define the *transitions*, it is necessary to establish the inner logic of M. Supposing that the cursor is standing on a chord while the machine remains in the state (degree) which matches it, we will acknowledge that the chord has been identified. Immediately after, M will perform two parallel tasks: it will fill the same position of the second and third tapes with the corresponding information (mode and function); and it will guess the previous chord in the succession, by moving the cursor leftward and changing the state to the applied diatonic fifth, its plagal preparation (only when the current degree is the first), its tritone substitution or that of its dominant, as long as one of the combinations coincides.

Otherwise, there is still a remaining possibility, which occurs when the previous chord X appears as a diatonic predecessor of some Y, but not in the current key – rather in the tonality of Y.[21] This implies that the sequence of chords between X and Y needs to be reassigned a mode and their functions. The procedure our automaton must conduct is the following: the cursor will stand still in the auxiliary tape, and it will move rightward and uniformly for the rest of tapes until the new tonic Y is found. Then, it will shift the cells on its right (by successively rewriting its neighbor while a state change preserves the information of the erased cell), so that it can create another instance of Y as a tonic, while preserving the older interpretation of Y as well. Afterwards, it will return to the modulating cell and modify the mode and the functions of the chords between X and Y accordingly.

This process works on a fair number of standards, but we still have not accounted for the return to the original key. To do so, given a chord X, we must give preference

[21] In practice, X usually the dominant of Y.

to the search for any subsequent chord which is related by a diatonic fifth with X (or by a plagal resolution, if it turns out to be the first degree of the mode). We would later try to locate a tritone substitution or a tonicization if the first attempt was unsuccessful.

Lastly, the functioning of M is interrupted if no viable choice is found, in which case, the automaton moves to an error state ε (where it either stops or it runs indefinitely). This scenario prevents us from reaching the accept state, which in normal conditions is accessed after the first chord has been parsed and the machine finds the initial symbol \$. Therefore, we can deduce the transitions from the collection of instructions given in the latest four paragraphs.

We will now use the resulting Turing machine M to analyze the progression $B\flat^{\Delta 7} - E\flat^{\Delta 7} - D^{-7} - G^{7} - C^{-7} - G\flat^{7} - F^{7} - B\flat^{\Delta 7}$, reminiscent of *My foolish heart*'s A section:

(1)	\$	**$B\flat^{\Delta 7}$**	$E\flat^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta 7}$	*	*
(2)	\$	*****	*	*	*	*	*	*	*	*	*
(3)	\$	*****	*	*	*	*	*	*	*	*	*
(4)	\$	*****	*	*	*	*	*	*	*	*	*

Current state: q_0

Figure 25: The automaton starts at the proper initial cell.

(1)	\$	$B\flat^{\Delta 7}$	**$E\flat^{\Delta 7}$**	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta 7}$	*	*
(2)	\$	*	*****	*	*	*	*	*	*	*	*
(3)	\$	*	*****	*	*	*	*	*	*	*	*
(4)	\$	$B\flat^{\Delta 7}$	*****	*	*	*	*	*	*	*	*

Current state: q_0

Figure 26: It then moves to the right cell and copies the previous chord, remaining in the initial state.

(1)	$	$B\flat^{\Delta 7}$	$E\flat^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta 7}$	*	*
(2)	$	*	*	*	*	*	*	*	*	*	*
(3)	$	*	*	*	*	*	*	*	*	*	*
(4)	$	$B\flat^{\Delta 7}$	$E\flat^{\Delta 7}$	*	*	*	*	*	*	*	*

Current state: q_0

Figure 27: The process is repeated...

(1)	$	$B\flat^{\Delta 7}$	$E\flat^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta 7}$	*	*
(2)	$	*	*	*	*	*	*	*	*	*	*
(3)	$	*	*	*	*	*	*	*	*	*	*
(4)	$	$B\flat^{\Delta 7}$	$E\flat^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	*	*	*

Current state: q_0

Figure 28: ... until the last chord is reached.

(1)	$	$B\flat^{\Delta 7}$	$E\flat^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta 7}$	*	*
(2)	$	*	*	*	*	*	*	*	*	*	*
(3)	$	*	*	*	*	*	*	*	*	*	*
(4)	$	$B\flat^{\Delta 7}$	$E\flat^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta 7}$	*	*

Current state: q_0

Figure 29: The automaton detects it after trying to move right to find another non-$*$ entry.

(1)	$	$B\flat^{\Delta 7}$	$E\flat^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta 7}$	*	*
(2)	$	*	*	*	*	*	*	*	*	*	*
(3)	$	*	*	*	*	*	*	*	*	*	*
(4)	$	$B\flat^{\Delta 7}$	$E\flat^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta 7}$	*	*

Current state: I

Figure 30: At this point, it moves leftward and changes to the first state I.

(1)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*
(2)	$	*	*	*	*	*	*	*	B♭	*	*
(3)	$	*	*	*	*	*	*	*	T	*	*
(4)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *V*

Figure 31: Since the previous chord is its dominant, it moves back and prints the information about the chord which it has left.

(1)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*
(2)	$	*	*	*	*	*	*	B♭	B♭	*	*
(3)	$	*	*	*	*	*	*	D	T		
(4)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *V/bV/V*

Figure 32: The same happens for the tritone substitution of a secondary dominant. The states also change accordingly.

(1)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*
(2)	$	*	*	*	*	*	C♭	B♭	B♭	*	*
(3)	$	*	*	*	*	*	D	D	T	*	*
(4)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *V/bV/V*

Figure 33: The auxiliary cursor points at the previous chord, but neither the main pointer nor the state are altered, since C^{-7} is not a diatonic fifth or a plagal preparation.

(1)	$	B♭$^{\Delta 7}$	E♭$^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta 7}$	*	*
(2)	$	*	*	*	*	*	C♭	B♭	B♭	*	*
(3)	$	*	*	*	*	*	D	D	T	*	*
(4)	$	B♭$^{\Delta 7}$	E♭$^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta 7}$	*	*

Current state: *V*

Figure 34: The main cursor goes back until it finds F^7, a diatonic fifth below.

(1)	$	B♭$^{\Delta 7}$	E♭$^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta 7}$	*	*
(2)	$	*	*	*	*	*	C♭	B♭	B♭	*	*
(3)	$	*	*	*	*	*	D	D	T	*	*
(4)	$	B♭$^{\Delta 7}$	E♭$^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta 7}$	*	*

Current state: *V*

Figure 35: The machine saves the state so that it can be related with the chord stored in the memory (C^{-7}).

(1)	$	B♭$^{\Delta 7}$	E♭$^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta 7}$	*	*
(2)	$	*	*	*	*	*	C♭	B♭	B♭	*	*
(3)	$	*	*	*	*	*	D	D	T	*	*
(4)	$	B♭$^{\Delta 7}$	E♭$^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta 7}$	*	*

Current state: *ii*

Figure 36: Now, C^{-7} can be interpreted as the *ii* degree of B♭ major.

(1)	$	B♭$^{\Delta 7}$	E♭$^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta 7}$	*	*
(2)	$	*	*	*	*	B♭	C♭	B♭	B♭	*	*
(3)	$	*	*	*	*	S	D	D	T	*	*
(4)	$	B♭$^{\Delta 7}$	E♭$^{\Delta 7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta 7}$	*	*

Current state: *ii*

Figure 37: Again, G^7 is not a proper chord of the tonality. Then, the main cursor remains in C^{-7}, which will be established as the new key.

(1)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	*	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*
(2)	$	*	*	*	*	*	C♭	B♭	B♭	*	*
(3)	$	*	*	*	*	*	D	D	T	*	*
(4)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *ii*

Figure 38: The information of C^{-7} is deleted (and stored in the memory as a state) so that the tonicalized interpretation of C^{-7} can be printed.

(1)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	*	C^{-7}	F^{7}	$B\flat^{\Delta7}$	*	*
(2)	$	*	*	*	*	*	B♭	B♭	B♭	*	*
(3)	$	*	*	*	*	*	S	D	T	*	*
(4)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *V/bV/V*

Figure 39: Now $G\flat^{7}$ is replaced by the saved chord, and the process is reiterated.

(1)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	*	C^{-7}	$G\flat^{7}$	$B\flat^{\Delta7}$	*	*
(2)	$	*	*	*	*	*	B♭	C♭	B♭	*	*
(3)	$	*	*	*	*	*	S	D	T	*	*
(4)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *V*

Figure 40: Still continuing...

(1)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	*	C^{-7}	$G\flat^{7}$	F^{7}	*	*
(2)	$	*	*	*	*	*	B♭	C♭	B♭	*	*
(3)	$	*	*	*	*	*	S	D	D	*	*
(4)	$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *I*

Figure 41: ... until an empty set of cells is found.

(1)	\$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	*	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*
(2)	\$	*	*	*	*	*	$B\flat$	$C\flat$	$B\flat$	$B\flat$	*
(3)	\$	*	*	*	*	*	S	D	D	T	*
(4)	\$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *I*

Figure 42: Here it will add the chord stored in the last place.

(1)	\$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	*	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*
(2)	\$	*	*	*	*	*	$B\flat$	$C\flat$	$B\flat$	$B\flat$	*
(3)	\$	*	*	*	*	*	S	D	D	T	*
(4)	\$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *i/ii*

Figure 43: The automaton returns to the chord in question.

(1)	\$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*
(2)	\$	*	*	*	*	C^{-}	$B\flat$	$C\flat$	$B\flat$	$B\flat$	*
(3)	\$	*	*	*	*	T	S	D	D	T	*
(4)	\$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *V/ii*

Figure 44: It repeats the transitions to add a dominant...

(1)	\$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*
(2)	\$	*	*	*	C^{-}	C^{-}	$B\flat$	$C\flat$	$B\flat$	$B\flat$	*
(3)	\$	*	*	*	D	T	S	D	D	T	*
(4)	\$	$B\flat^{\Delta7}$	$E\flat^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	$G\flat^{7}$	F^{7}	$B\flat^{\Delta7}$	*	*

Current state: *ii/ii*

Figure 45: ... and a diatonic fifth.

(1)	$	B♭$^{\Delta7}$	E♭$^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta7}$	*
(2)	$	*	*	C^{-}	C^{-}	C^{-}	B♭	C♭	B♭	B♭	*
(3)	$	*	*	S	D	T	S	D	D	T	*
(4)	$	B♭$^{\Delta7}$	E♭$^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta7}$	*	*

Current state: *I*

Figure 46: Since E♭$^{\Delta7}$ is not related to the chords, the cursor moves right until it finds a plagal resolution in the original key (which state is saved).

(1)	$	B♭$^{\Delta7}$	E♭$^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta7}$	*
(2)	$	*	*	C^{-}	C^{-}	C^{-}	B♭	C♭	B♭	B♭	*
(3)	$	*	*	S	D	T	S	D	D	T	*
(4)	$	B♭$^{\Delta7}$	E♭$^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta7}$	*	*

Current state: *IV*

Figure 47: The machine returns to E♭$^{\Delta7}$ to attribute it a *IV* degree.

(1)	$	B♭$^{\Delta7}$	E♭$^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta7}$	*
(2)	$	*	B♭	C^{-}	C^{-}	C^{-}	B♭	C♭	B♭	B♭	*
(3)	$	*	♭S	S	D	T	S	D	D	T	*
(4)	$	B♭$^{\Delta7}$	E♭$^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta7}$	*	*

Current state: *I*

Figure 48: And the last chord is deduced from a diatonic fifth transformation.

(1)	$	B♭$^{\Delta7}$	E♭$^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta7}$	*
(2)	$	B♭	B♭	C^{-}	C^{-}	C^{-}	B♭	C♭	B♭	B♭	*
(3)	$	T	♭S	S	D	T	S	D	D	T	*
(4)	$	B♭$^{\Delta7}$	E♭$^{\Delta7}$	D^{-7}	G^{7}	C^{-7}	G♭^{7}	F^{7}	B♭$^{\Delta7}$	*	*

Current state: q_F

Figure 49: Finally, the information of the initial B♭$^{\Delta7}$ is printed and the automaton moves to the final state after the tape reaches $.

The most remarkable peculiarity of this analysis is the connection between the IV degree and the initial tonic: it already raised some doubts during the previous study of the complete tune, but the fact that such unalike conclusions can be drawn highlights a shortage of specificity in our proposed models. However, the automaton has been able to retain the defining features of the explored systems: the left-branching nature of Rohrmeier's syntax and the assignation of tonal functions from embedded structural modes. Even if the body of works which could be analyzed by this construction is limited and gives room to ambiguity, we can easily extract a computer-coded implementation from it to further study its limitations.

Chapter 5

Conclusions

This research project has shown an eclectic compendium of views on tonal and modal jazz harmony, in the form of several practical methods stemming from two central models and ways to understand chordal material. Even if this in itself can prove as a worthwhile source of inspiration in creative or interpretative terms, we must evaluate our undertaking with respect to the initial objectives.

We have been capable of inferring some of the most significant assumptions which both systems make regarding harmonic motion within tonality or classic modality: most of them either refer to the structure (which involves larger momenta and issues about repetitions and continuity) or to local interactions between chords (inside a single mode or between two of them). However, most of them ultimately circled back to the core suppositions of each model, such as fifth-based motion in the case of embedded structural modes, or left-branching hierarchies for the generative grammar. But these were enough as a basis for the forthcoming algorithms and constructions.

For instance, the structural modes' apparent lack of a direct means of expressing higher instances of formal elements was addressed, as hinted by the authors, by introducing a recursive resource which was already a central element of Rohrmeier's syntax. This intersection between the axiomatizations spurred the possibility of considering an inverse perspective, that is, the elaboration of grammars and automata based on the syntactic approach, which could involve notions from fundamental basses. Finally, this led to a –functional– prototype of a mixed model which is not closed, in the sense that it could assimilate more data.

The search for more powerful and representative formalizations had its origin in their contextualization within a theoretical context. It is only natural to ask oneself if a slightly different mathematical object (possibly an generalization or a restricted version) can process the inputs in question in an improved way, with reference to the research goals. Also, having a correlative of an object grounded in a formal theory allows for a variety of equivalence results, which –in the case of grammars– facilitate the implementation in computer programs.

On the other hand, from a practical mindset, the deduction of conclusions from the application of the proposed analytical methods hinted at an approximate outline about how both models operate. For instance, the local behavior of the generative grammar pinpoints the pitch of the bass note for every production rule, while the double interpretation of horizontal arrows in structural modes could lead to some ambiguities depending on the modal context (as long as their meaning is not made elucidated). Nevertheless, it is in the consideration of larger structures where Rohrmeier's proposal leaves room to ambiguity and weaker connections, whereas –in the case of embedded structural modes– the inclusion of meta-modes allows us to define layers which can be used as a tool to interpret the tune linearly in different levels of complexity.

Needless to say, these intrinsic features (which some could be tempted to deem limitations) are actually a cornerstone for the expressiveness of the models: the tree graphs produced by the syntax depict in a very illustrative manner the understanding of the analyst about tension-resolution frameworks; and the arrow notation provides an accessible way to interpret the motion of the bass in relation to the overall form.

Moreover, such a degree of flexibility is beneficial for research enterprises in that we can introduce variations and different implementations without compromising the core foundations of the systems. Along these lines, we can consider the possibility to compare the results of an eventual coded version of the models with the outcome of a Markovian computation. Similarly, if we were to recover one of the presented automata, we could analyze its resulting accepted language in order to notice which assumptions from the base models still seem to hold or be required.

This task is inherently tied to another suggestion for further work: first, some sort of additional layer of information could be added to the code in order to illustrate better the generating process, similar to (if not) figure 23. Furthermore, one could insert some of the production rules which were not considered in the previous prototypes.

Overall, the comparative analysis of systems for harmonic parsing constitutes a synthetic method of research which offers a positive outlook both in a theoretical and applied level. It is compelling to envisage the possibilities which could result from considering another two different models.

Appendices

As a quick reference, we include a version of the lead sheet of each standard which we studied during this work [Hal04]. In case the reader's PDF viewer does not support double-page presentation, we also attach the full synctactic trees from section 3.1.

Figure 50: Lead sheet of *Autumn leaves*

Figure 51: Lead sheet of *Autumn leaves* (variation)

Figure 52: Lead sheet of *There will never be another you*

Figure 53: Lead sheet of *Blue in green*

Figure 54: Lead sheet of *Round midnight*

Figure 55: Lead sheet of *Stella by starlight*

Figure 56: Lead sheet of *My foolish heart*

Figure 57: Lead sheet of *Giant steps*

Figure 58: Lead sheet of *Orbit*

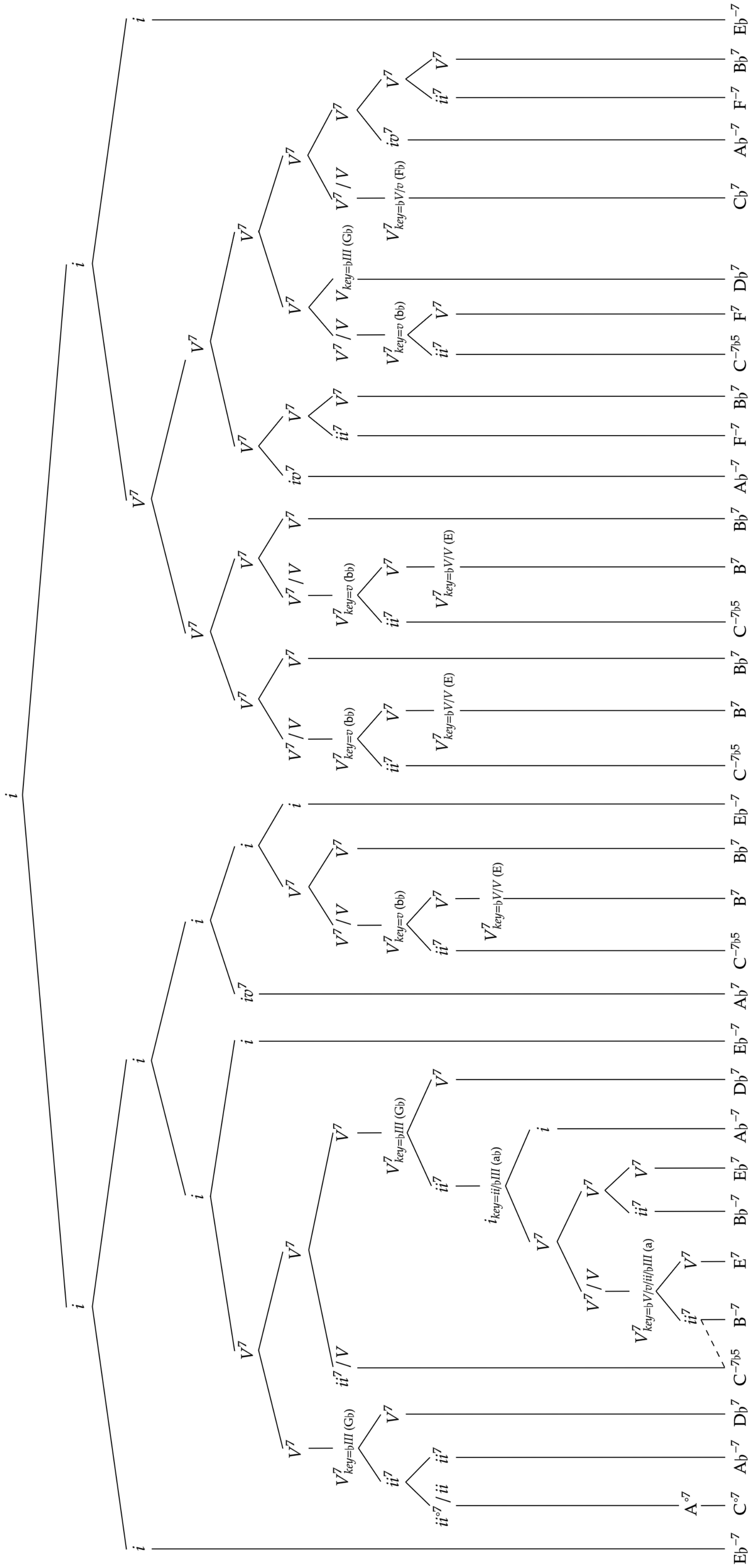

Figure 59: Full tree for *Round midnight*.

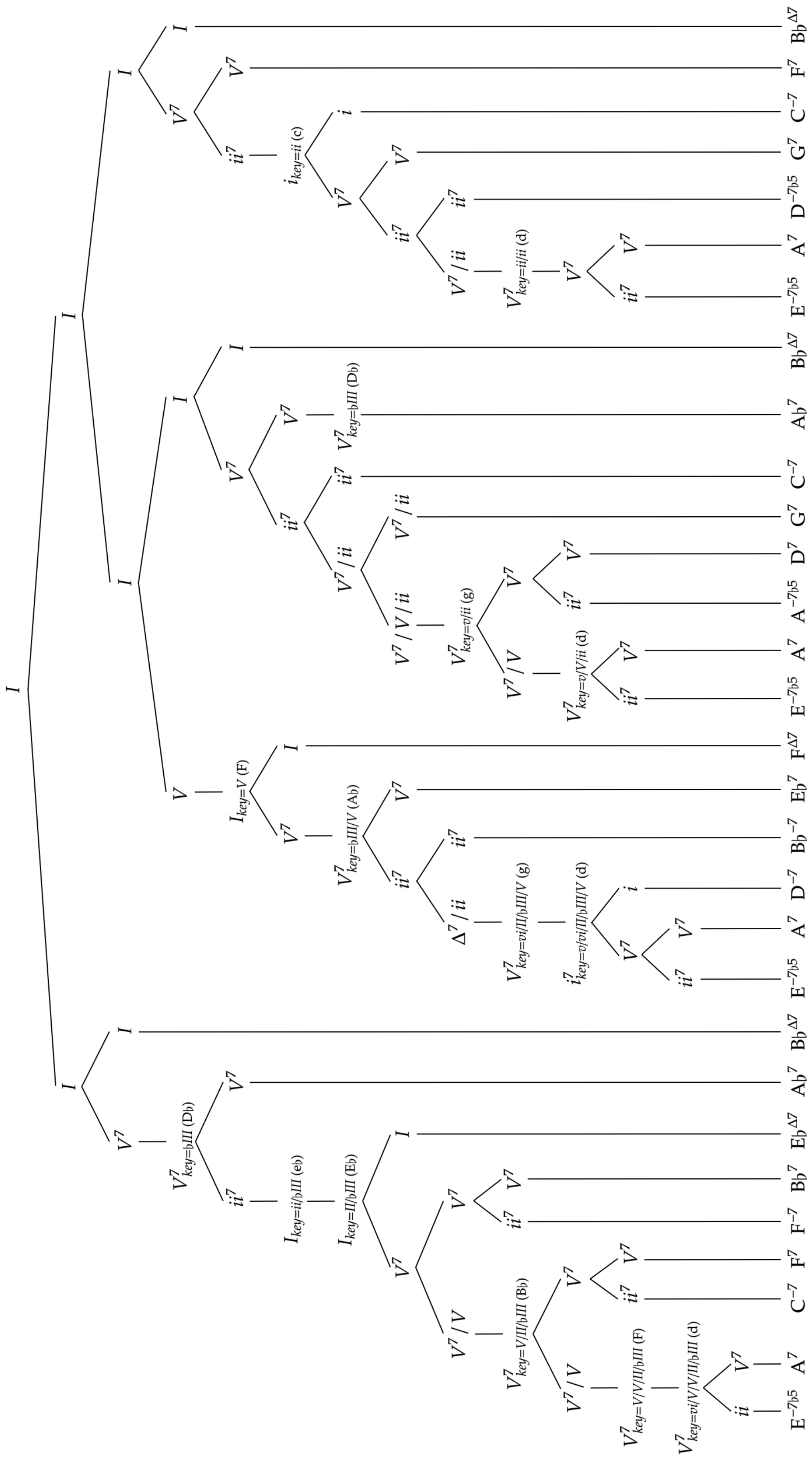

Figure 60: Full tree for *Stella by starlight*.

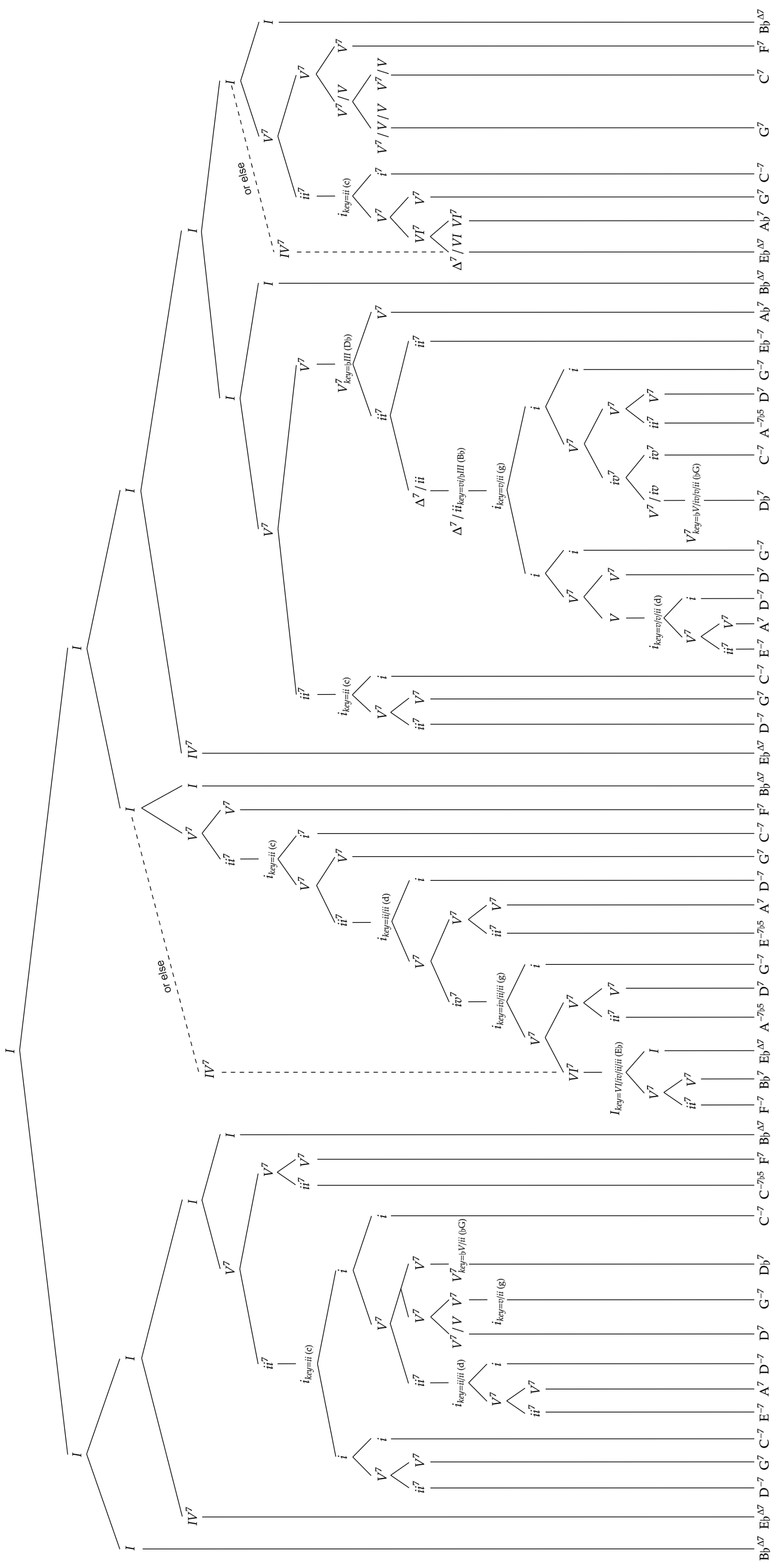

Figure 61: Full tree for *My foolish heart*.

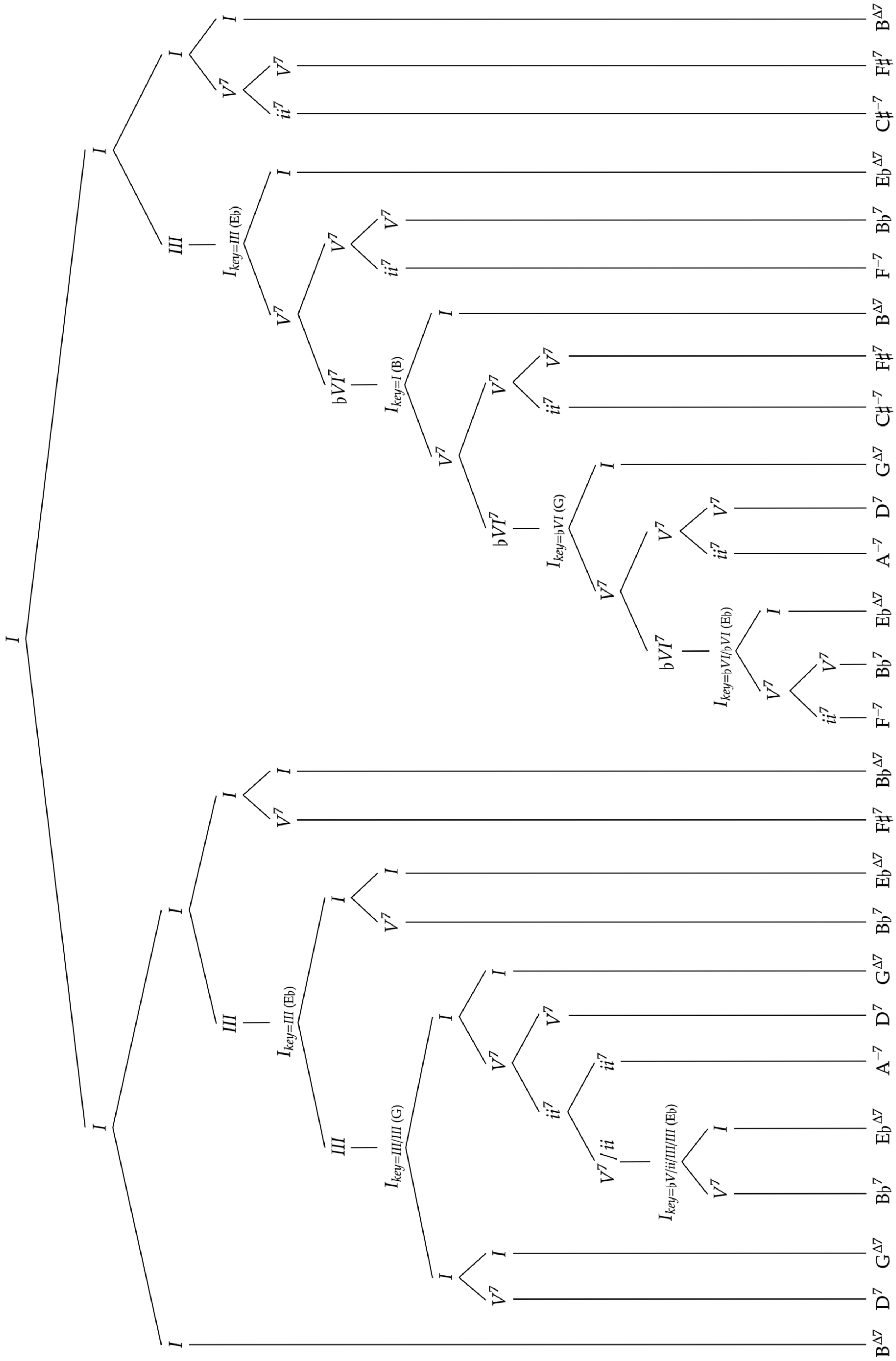

Figure 62: Full tree for *Giant steps*.

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