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When do central banks prefer to intervene secretly?

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Abstract

Central banks often intervene secretly in the foreign exchange market. This secrecy seems to be at odds with the signalling channel. In this article we will analyse when a central bank intervening in the foreign exchange rate market purely through the signalling channel would prefer to act secretly or publicly. By using a microstructure model, we will show that the consistency of the intervention with fundamentals, the volume of noise trading, the weight given to the effectiveness of intervention and the degree of superior information held by the central bank will influence the decision to intervene secretly or publicly.

Keywords: foreign exchange intervention, market microstructure

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1 Introduction

The secrecy surrounding central banks' sterilised intervention in the foreign exchange market has been puzzling. In particular, the tendency to intervene secretly has been at odds with one of the channels through which the central bank can alter exchange rates, the signalling channel. According to this channel, the central bank can influence exchange rate expectations through the provision of superior information or a signal about the future exchange rate to the market. If the objective of the intervention is to provide a signal, why would central banks conceal their intentions?

Dominguez and Frankel (1993) offer several possibilities about the role of secrecy in foreign exchange intervention. One such possibility relates to central bank's convictions about the efficacy of the intervention. Thus, when intervention is regarded as inconsistent with fundamentals, the monetary authority might prefer not to draw attention to the intervention.¹ Also, when the decision to intervene comes from another authority, the central bank might be inclined to conduct the intervention secretly. This case arises when a central bank, like the Bank of Japan, plays the role of agent of the Ministry of Finance. In these circumstances, a mismatch in the objectives of these institutions may arise. For instance, the objectives of the central bank might be related to economic fundamentals whereas those of the Ministry of Finance might involve achieving exchange rate levels that are more favourable for exports.²

Given the extensive empirical literature on the determinants of central bank intervention,³ it is somehow surprising that the choice between secret and public intervention has

¹International economics theories state that the exchange rate is determined by macroeconomic fundamentals such as money supplies, outputs and interest rates. The fundamental value of the exchange rate is thus known as fundamental(s). See, for instance, Engel and West (2005).

²According to article 7 of The Foreign Exchange and Foreign Trade Law, the Ministry of Finance shall endeavour to stabilise the external value of the yen through foreign exchange trading.

³See, for instance, Sarno and Taylor (2001) for a survey.

hardly been studied. An exception are Beine and Bernal (2007) and Beine et al. (2007), who study the case of forex interventions in Japan. Beine and Bernal (2007), using a simple logit model, find that the Bank of Japan tended to favour the use of secret interventions 35% more often when the intervention was inconsistent, aiming at increasing rather than reducing the degree of misalignment of the yen. Beine et al. (2007), using a nested logit approach, find the opposite result.⁴

The secrecy of intervention has been justified theoretically through microstructure models. These models are ideal candidates to analyse central banks' interventions, particularly as empirical evidence by Peiers (1997), Chang and Taylor (1998) and Dominguez (2003) shows that the market has learnt about central bank intervention through trading rather than public news shown on the Reuters news screen. At the theoretical level, Bhattacharya and Weller (1997) analyse the secrecy of interventions when the central bank does not have privileged information about the fundamental with respect to price takers. Vitale (1999), on the other hand, uses a model based on Kyle (1985), where the central bank has superior information about the fundamental. Recently, Barnett and Ozerturk (2007) analyse the case where a central bank can disclose relevant information to another central bank but not to the rest of the market participants. These three articles share the characteristic that the central bank's target is not related to the fundamentals and justify the secrecy of intervention. However, does this mean that inconsistent central bank interventions are always secret? Further, central bank interventions are sometimes consistent with fundamentals.

⁴Beine et al. (2007) explain that the different result obtained in Beine and Bernal (2007) might be due to a failure to decompose the decision process into sequential intervention steps. In Beine et al. (2007), it is recognised that forex intervention is a complex process with several dimensions (determinants of intervention, the choice of intervention strategy and the market's perception) that are interdependent, and an estimation through a nested logit describing the various steps allows for this interdependence to be taken into account.

For Mussa (1981), central bank intervention should be aimed at revealing the fundamental value of the exchange rate. Besides, there is a broad consensus among central bank's authorities that consistency of intervention with fundamentals is important (see Neely's (2006) survey). Then, when interventions are consistent with fundamentals, will they be public or secret?

In this article we will analyse when a central bank intervening in the foreign exchange rate market purely through the signalling channel would prefer to act secretly or publicly. Further, in order to address the mixed results provided by the empirical literature, we will also study whether the distinction between consistency and inconsistency of the central bank's target with fundamentals will make a difference in this decision. To that end, the model to be used must also allow for the central bank to be "consistent", i.e., to give information about the future that is related to the fundamentals and, therefore, we will include in our model a relationship between the fundamentals and the central bank's target.

We will show that there are a series of factors that will determine whether a central bank prefers to carry out sterilised foreign exchange rate intervention secretly or publicly. Among the factors, we find that the consistency or inconsistency of the intervention, the volume of noise trading, the weight given to the effectiveness of intervention and the degree of superior information held by the central bank contribute to that decision. Further, we will also demonstrate that secret intervention can be an optimal choice even when such intervention is consistent with fundamentals.

The remainder of this paper is organized as follows. Section 2 presents the model, notation and hypotheses for the two scenarios considered: the non-secret setup (common knowledge) and the secret one. Section 3 states the unique linear equilibrium corresponding to both settings and compares some market indicators. In Section 4 we analyse and compare

the loss function of the central bank under common knowledge and under secrecy, and show under what circumstances it will prefer to intervene publicly or secretly. Concluding comments are presented in Section 5. Finally, the proof of the main results is included in the Appendix.

2 The model

In this section we describe the basic structure of our model, which is similar to Kyle (1985) and Vitale (1999). We consider an economy where a single foreign currency is traded at an exchange rate s_1 . Its fundamental value is represented by f , which is normally distributed. Three kinds of agents participate in the foreign exchange market: the central bank, noise traders and market makers.

The central bank acts strategically and it perfectly knows the value of f because it possesses superior information. Given that we will consider sterilised central bank interventions, the value of the fundamental will not be affected by such interventions. The central bank will also have a target level for the exchange rate, denoted by t .⁵ In order to reflect both the possibility of interventions that are "consistent" with the fundamental and interventions that are "inconsistent", we assume that the fundamental value and the target level are dependent, and in particular, $t = f + \eta$, where η represents a term not related to the fundamental. Thus, in our case, when intervention by the central bank is consistent, the target value for the exchange rate will be completely related to the fundamental value and thus the variance of η will tend to zero. On the other hand, when the intervention is inconsistent, the target value of the exchange rate will not be related to fundamentals and

⁵This is in accordance with empirical evidence. For instance, Frenkel et al. (2003) provide evidence that the Bank of Japan had a target of 125 yen/Us dollar in the '90s.

the variance of η will tend to infinite. In this limiting case, our model would be close to Vitale's (1999) model.

The literature mentions varied goals for central banks' intervention in the foreign exchange market. For instance, Neely (2001) and Chiu (2003), in surveys about the practice of central bank intervention, report that monetary authorities often employ intervention to resist short-run trends in exchange rates and also to return exchange rates to fundamentals. Thus, according to these objectives, the central bank might have a target for the exchange rate that in occasions is close to fundamentals and in occasions it is not. Further, Neely states that profitability is a useful gauge of central banks' success as careful stewards of public resources. On the lines of this empirical evidence, in our model the central bank will choose its market order, denoted by x , in order to minimize the expected value of its loss function, which includes a first term that reflects the speculative intervention, and a second term that reflects the stabilization of the exchange rate around the target:⁶

$$L = (s_1 - f)x + q(s_1 - t)^2. \quad (1)$$

The parameter q represents the weight given by the central bank to exchange rate stabilization. In particular, when the central bank's motives are purely speculative, $q = 0$. On the contrary, when the focus of its intervention is achieving the target, then q will be large.

The second type of agents are noise traders, also known as liquidity traders, who demand a random, inelastic quantity, denoted by ε . Their demand is not based on maximizing behavior.

The third type of agents are market makers. This is in accordance with the tradition of

⁶This loss function is now generally accepted in the analysis of central bank intervention in the foreign exchange market (see, for instance, Bhattacharya and Weller (1997) and Vitale (1999), among others). Note that when choosing its market order, the central bank observes f and t , but not ε .

central bank interventions, particularly in the mid-1980s and 1990s, when most G-7 central banks dealt directly with bank dealers rather than brokers.⁷ Market makers observe the aggregate order flow quantity $x+\varepsilon$ and set the exchange rate at which they trade the quantity necessary to clear the market.⁸ We assume market makers are risk neutral. Competition among market makers forces them to choose the exchange rate such that they earn zero expected profits. Thus, we have

$$s_1 = E(f|I_m),$$

where I_m represents the information set of the market makers.

All random variables are assumed to be normally distributed. More precisely, $f \sim N(s_0, \sigma_f^2)$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and $\eta \sim N(e - s_0, \sigma_\eta^2)$,⁹ where e represents the unconditional expected value of the target level, t . Furthermore, it is supposed that these random variables are independent of each other. The joint distribution of all these random variables is common knowledge.

3 Equilibrium

In this section we derive the equilibrium exchange rate when the central bank intervenes publicly and secretly. We will follow the terminology of Vitale (1999), and will refer to

⁷See, for instance, Humpage (1994) and Dominguez and Frankel (1993). Neely's survey (2001) reports that monetary authorities have some preference for dealing with major domestic (and foreign) banks.

⁸Alternatively to the existence of noise traders, we could consider that central bank orders are routed through an individual dealer. The subsequent trades of that dealer could be interpreted as noisy signals of the intervention, given the low transparency of foreign exchange markets. See Vitale (1999) and Lyons (2001) for more on this issue.

⁹The assumption of normality implies that the variance of the noise trading is positively related to the volume of noise trading. In the text, we will use the terms variance of noise trading and volume of noise trading indistinctly.

the case where the central bank makes public its target for the exchange rate as "common knowledge" and the case where it keeps it concealed as "secrecy".

3.1 Common knowledge of the target level

We initially assume that market makers are able to observe the target level of the central bank. The following result derives the unique linear equilibrium in this framework:

Proposition 1: *If the target level is common knowledge, there exists a unique linear Nash equilibrium defined as follows:*

$$s_1 = E(f|x + \varepsilon, t) = s_0 + 2\lambda q (s_0 - t) + \frac{4(q\lambda + 1)^2 \sigma_\varepsilon^2 \lambda^2}{\sigma_\eta^2} (t - e) + \lambda(x + \varepsilon)$$

and

$$x = \beta(f - s_0) + \gamma(t - s_0) + \delta(t - e),$$

where λ is the unique positive root of the following polynomial:

$$g(\lambda) = 4\lambda^2 (q\lambda + 1)^2 \sigma_\varepsilon^2 (\sigma_f^2 + \sigma_\eta^2) - (2q\lambda + 1) \sigma_f^2 \sigma_\eta^2,$$

with $\beta = \frac{1}{2\lambda(q\lambda+1)}$, $\gamma = 2q$ and $\delta = -\frac{2(2q\lambda+1)(q\lambda+1)\sigma_\varepsilon^2 \lambda}{\sigma_\eta^2}$.

It is interesting to notice that this proposition indicates that the equilibrium coefficients depend on σ_η^2 . In Vitale's (1999) framework, where the target level is independent of the fundamental value, market makers only use the target level to disentangle part of the noise of the order flow. By contrast, when the target level is correlated with the fundamental value, as it is in this model, then the target level is not only helpful to disentangle part of the noise of the order flow, but it is also useful to provide information about the fundamental value.

3.2 Secrecy of the target level

We now assume that market makers do not know the exact value of the target level of the central bank. The unique linear equilibrium in this case is shown in the following result:

Proposition 2: *If the target level is secret, there exists a unique linear Nash equilibrium defined as follows:*

$$s_1 = E(f|x + \varepsilon) = s_0 + \lambda(x + \varepsilon - 2q(e - s_0))$$

and

$$x = \beta(f - s_0) + \theta(t - e) + \gamma(e - s_0),$$

where λ is the unique positive root of the following polynomial:

$$h(\lambda) = 4\lambda^2 q^2 \sigma_\eta^2 + 4\lambda^2 (q\lambda + 1)^2 \sigma_\varepsilon^2 - (2q\lambda + 1) \sigma_f^2,$$

with $\beta = \frac{1}{2\lambda(q\lambda+1)}$, $\gamma = 2q$ and $\theta = \frac{q}{(q\lambda+1)}$.

In contrast to Proposition 1, it can be seen that now the equilibrium exchange rate does not depend directly on t , given that this information is not observed by market makers. Furthermore, Propositions 1 and 2 indicate that in both scenarios the endogenous variable λ shows how useful is the order flow in the forecast of the fundamental value. Notice that under common knowledge, the order flow should be considered as a "net" order flow, as market makers have been able to disentangle part of the noise of the order flow thanks to the knowledge of t .

>From now on, we will use a superscript in a variable to represent the framework in which this variable is obtained. Thus, for instance, λ^{CK} will refer to the variable λ in the framework of common knowledge of the target level, while λ^S will represent this variable when the target level is secret.

3.3 Effects of the intervention choice on market indicators

It is interesting to study and compare the characteristics of the market that arise under each of the two equilibria obtained. Traditionally, the literature on market microstructure has analysed two market indicators: market liquidity and efficiency.

Following Kyle (1985), the market liquidity is measured by the inverse of the endogenous variable λ , since this inverse represents the order flow necessary to induce the exchange rate to rise or fall by one unit. Therefore, the higher the market liquidity, the lower will be the impact of the order of one particular agent on the price of the currency. On the other hand, a usual measure of the market efficiency is the inverse of the conditional variance of the fundamental value given the information set of market makers, that is, $var^{-1}(f|I_m)$. The following corollary provides the results of the comparison of market liquidity and efficiency under secrecy and under common knowledge:

Corollary 1: *The foreign exchange market is more -or at least equally- liquid and efficient when the central bank conceals its target level if and only if*

$$\sigma_\varepsilon \leq \frac{2\sigma_\eta^4 q}{\sigma_f (\sigma_f^2 + 2\sigma_\eta^2)}. \quad (2)$$

This corollary shows that there is a direct relationship between market liquidity and efficiency. However, it points out that only when (2) holds, the market liquidity and efficiency are higher under secrecy. Vitale (1999) and Barnett and Ozerturk (2007) find that, when the target level and the fundamental value are independent, the foreign exchange market is always more liquid and efficient under secrecy. Note that Corollary 1 includes these results (when $\sigma_\eta^2 \rightarrow \infty$), but it also covers cases where the common knowledge of the target will provide higher liquidity and efficiency: when σ_η^2 is small, σ_ε^2 is large, q is small and σ_f^2 is large.

The above result is interesting given that the properties of liquidity and efficiency are obviously of interest for central banks. To illustrate the effects of common knowledge on market liquidity, consider first the case where σ_η^2 is small, that is, the target of the central bank is fairly close to the fundamental. Under common knowledge, the target level is almost sufficient to forecast the fundamental value, and thus, the order flow provides little additional information to set the price of the currency,¹⁰ making market liquidity almost infinite.

Second, when σ_ε^2 is large enough, even though the order flow is a very noisy signal of the fundamental value, when market makers also observe the target level, the forecast of the fundamental value in this case is almost exclusively based on the target level, and thus market liquidity is higher in the common knowledge setup.

Third, when q is small, the main motive of intervention for the monetary authority is speculative. The central bank realizes that in the more transparent market structure -i.e., when the target level is common knowledge- it cannot act so aggressively since otherwise it would reveal too much private information. Market makers are aware of this and therefore they set a smaller λ .

Finally, when σ_f^2 is large, that is, there is more asymmetric information as the central bank perfectly knows f , the target value is more correlated with the fundamental value. Under common knowledge, the market makers' prediction about the fundamental value will rely more on the target and less on the order flow. Hence, the market liquidity will be, again, higher under common knowledge than under secrecy.

In relation to market efficiency, at first glance one would expect to find that the common knowledge setup is more efficient since in this framework, when setting the exchange rate, market makers observe the order flow and the target level, while in the secret setup they only

¹⁰Recall that λ shows how useful is the order flow in the forecast of the fundamental value and hence, in this case λ is very small.

observe the order flow. An illustrative case where this result holds is when σ_η^2 converges to zero. In this case, under common knowledge market makers are able to completely deduce f from the target level. Hence, $\text{var}(f|I_m^{CK})$ tends to zero, and therefore, the measure of market efficiency goes to infinity. By contrast, in the secret setup market makers do not perfectly infer f from the order flow and the measure of efficiency is finite.

Corollary 1 shows that the opposite result may, however, hold because of the strategic behavior of the central bank. For instance, when σ_η^2 tends to infinity, the central bank must intervene cautiously under common knowledge since otherwise it would reveal too much private information. When this situation arises, the secret setup is more efficient.¹¹

In order to have further insight into the relationship between market liquidity and the different parameters of the model, we present some comparative statics results for λ under both scenarios, common knowledge and secrecy of the target level in the following corollaries:

Corollary 2: *If the target level is common knowledge,*

a) λ^{CK} increases in σ_η^2 . When σ_η^2 converges to zero, λ^{CK} tends to zero. In the opposite case, when σ_η^2 converges to infinity, λ^{CK} converges to $\bar{\lambda}$, which is the unique positive value that satisfies the following equation:

$$4\lambda^2 (q\lambda + 1)^2 \sigma_\varepsilon^2 - (2q\lambda + 1) \sigma_f^2 = 0. \quad (3)$$

b) λ^{CK} decreases in σ_ε^2 . When σ_ε^2 converges to zero, λ^{CK} tends to infinity. In the opposite case, when σ_ε^2 converges to infinity, λ^{CK} tends to zero.

c) λ^{CK} decreases in q . When q converges to zero, λ^{CK} tends to $\frac{\sigma_f \sigma_\eta}{2\sigma_\varepsilon \sqrt{\sigma_\eta^2 + \sigma_\eta^2}}$. In the opposite case, when q converges to infinity, λ^{CK} tends to zero.

d) λ^{CK} increases in σ_f^2 . When σ_f^2 tends to zero, λ^{CK} converges to zero. In the opposite

¹¹A similar result has also been found in other microstructure models where quantity of information is not equivalent to quality of information (see, for instance, Manzano (2002)).

case, when σ_f^2 tends to infinity, λ^{CK} converges to the unique positive value that satisfies the following equation:

$$4\lambda^2 (q\lambda + 1)^2 \sigma_\varepsilon^2 - (2q\lambda + 1) \sigma_\eta^2 = 0.$$

Corollary 3: *If the target level is secret,*

a) λ^S decreases in σ_η^2 . When σ_η^2 converges to zero, λ^S converges to $\bar{\lambda}$, which is the unique positive value that satisfies (3). In the opposite case, when σ_η^2 converges to infinity, λ^S tends to zero.

b) λ^S decreases in σ_ε^2 . When σ_ε^2 converges to zero, λ^S tends to $\frac{\sigma_f^2 + \sqrt{\sigma_f^4 + 4\sigma_f^2 \sigma_\eta^2}}{4q\sigma_\eta^2}$. In the opposite case, when σ_ε^2 converges to infinity, λ^S tends to zero.

c) λ^S decreases in q . When q converges to zero, λ^S tends to $\frac{\sigma_f}{2\sigma_\varepsilon}$. In the opposite case, when q converges to infinity, λ^S tends to zero.

d) λ^S increases in σ_f^2 . When σ_f^2 tends to zero, λ^S converges to zero. In the opposite case, when σ_f^2 tends to infinity, λ^S converges to infinity.

Notice the different pattern of λ with respect to the variance of η in the two frameworks compared. An increase in σ_η^2 , under secrecy, renders the order flow noisier and less informative about the fundamental, turning the price of the currency less sensitive to the order flow. Under common knowledge, on the other hand, as the target is less related to the fundamental with a higher σ_η^2 , market makers rely more on the net order flow when setting the price of the currency. Therefore, liquidity increases with σ_η^2 under secrecy and decreases under common knowledge.

The relationship obtained in Corollaries 2 and 3 between the variance of the volume of noise trading (σ_ε^2) and the level of commitment to the target level of the central bank (q) with market liquidity is similar to the one derived by Vitale (1999). The market liquidity is increasing in the variance of noise trading, independently of the market makers' knowledge

about the target level. The economic intuition for this result is similar to the one in Kyle (1985): an increase in liquidity trading reduces the adverse selection problem of market makers, and this leads to a decrease in λ . Further, the market liquidity is increasing in the level of commitment to the target level of the central bank, independently of the market makers' knowledge about the target level. An increase in q reduces the monetary authority's incentives to exploit its private information to make profits, and this decreases the informativeness of the central bank's order, which explains the reduction in λ .

Finally, Corollaries 2 and 3 show that in both scenarios there is a direct relationship between λ and the variance of the fundamental. Notice that a rise in σ_f^2 increases the adverse selection problem of market makers as there is more asymmetry in the information held by the central bank with respect to the rest of the market. Consequently, this leads to an increase in the value of λ set by market makers.

4 Deciding whether to intervene secretly or publicly

The central bank will decide whether to intervene secretly or publicly according to the expected value of its loss function under each scenario. Recall that the losses of the central bank, given in Equation (1), consist of two terms, the first one related to the cost of trading and the second one related to the stabilisation of the exchange rate. We will study these terms in the next two subsections.

4.1 Cost of trading

The first term of the loss function of the central bank indicates, as shown in the next lines, whether the central bank intervention is expected to be profitable. Using the fact that, in equilibrium, competitive risk-neutral market makers obtain zero expected profits, it follows

that in both scenarios

$$E((s_1 - f)x) = -E((s_1 - f)\varepsilon) = -\lambda\sigma_\varepsilon^2, \quad (4)$$

a result that is standard in microstructure models *à la* Kyle (1985). From this expression we obtain that the cost of trading of the central bank is negative, which would indicate profits. This is due to the fact that the central bank has superior information about the fundamental. The possibility that central banks have at least broken even on floating exchange rate intervention or that they have made large profits has received empirical support by the more recent studies on profitability, like those of Saacke (2002), Sjöö and Sweeney (2001), Sweeney (2000) and Leahy (1995).

As it can be seen from Expression (4), the cost of trading is related with the endogenous variable λ and, as mentioned before, its inverse is a measure of market liquidity. Further, there is a positive relationship between market liquidity and the cost of trading. Therefore, the analysis of the cost of trading is reduced to the study of market liquidity, which was performed in Corollary 1. According to this corollary, market liquidity will be higher under secrecy as long as the variance of η is large, the volume of noise trading is low, the weight placed by the central bank on stabilisation is large and when there is less asymmetric information. In these circumstances, as market liquidity will be higher under secrecy, the cost of trading will also be higher.

In order to see how the consistency or inconsistency of intervention will affect the cost of trading, it is helpful to notice that Corollaries 2 and 3 show that the relationship between λ and σ_η^2 follows a very different pattern under common knowledge and under secrecy. If the intervention is consistent ($\sigma_\eta^2 = 0$), the cost of trading of the central bank under common knowledge is zero, and under secrecy is $-\bar{\lambda}\sigma_\varepsilon^2$, thus obtaining an expected profit only in the last case. By contrast, when the intervention is not consistent ($\sigma_\eta^2 \rightarrow \infty$), the cost

of trading of the central bank under common knowledge is $-\bar{\lambda}\sigma_e^2$, and under secrecy is zero. In summary, these results indicate that under either type of intervention, secret or common knowledge, there is a trade-off between liquidity and profitability: when the central bank's intervention is consistent with fundamentals, the secrecy of the target will provide lower liquidity and an expected profit; on the contrary, when there is no consistency with fundamentals, intervening publicly will be associated with lower liquidity and an expected profit.

4.2 Effectiveness

The effectiveness of the central bank will depend upon how successful its intervention is in bringing the exchange rate close to the target. Therefore, we measure the effectiveness of the central bank's intervention as $\frac{1}{E((s_1-t)^2)}$, which is related to the second term of the loss function. The more effective the central bank expects to be, the closer the exchange rate will be to the target value, and hence, our measure of effectiveness will be higher.¹² The following result provides the expression for $E((s_1-t)^2)$ in the two scenarios that we compare and the result for their effectiveness:

Corollary 4: *If the target level is common knowledge, then*

$$E\left((s_1^{CK} - t)^2\right) = \frac{\left(\sigma_f^2 + 2\left(q\lambda^{CK} + 1\right)\sigma_\eta^2\right)\sigma_\eta^2}{2\left(q\lambda^{CK} + 1\right)\left(\sigma_f^2 + \sigma_\eta^2\right)} + (s_0 - e)^2, \quad (5)$$

while if it is secret, then

$$E\left((s_1^S - t)^2\right) = \frac{\sigma_f^2 - 2\left(q\lambda^S - 1\right)\sigma_\eta^2}{2\left(q\lambda^S + 1\right)} + (s_0 - e)^2. \quad (6)$$

¹²Bhattacharya and Weller (1997) analyse the effectiveness by looking at the limit $(s_1 - t)$, which is an ex-post measure. Our measure is an ex-ante one, before any realisation has been observed, and therefore shows the effectiveness the central bank expects to attain through its intervention.

Moreover, the central bank's effectiveness is higher -or at least equal- under secrecy if and only if (2) holds.

According to Corollary 4, the effectiveness of the central bank intervention will be higher under secrecy as long as the variance of η is large, the volume of noise trading is low, the weight placed by the central bank on stabilisation is large and when there is less asymmetric information. It is interesting to note that Barnett and Ozerturk (2007) also find that if the market's uncertainty over the central bank's target is sufficiently low, selectively disclosing the exchange rate target could decrease the effectiveness of intervention over secrecy.

Further, under a consistent policy ($\sigma_\eta^2 = 0$), the central bank is more effective when the target level is common knowledge. In this framework market makers can completely derive the fundamental value from the target level, and hence, they set $s_1^{CK} = f$. Therefore, s_1^{CK} and t may only differ by a constant. By contrast, under an inconsistent policy ($\sigma_\eta^2 \rightarrow \infty$), the central bank is completely ineffective both in secrecy and under common knowledge (i.e., $\frac{1}{E((s_1 - t)^2)} \rightarrow 0$ in both cases).

4.3 Losses

Drawing on the preceding subsections, we have found that when Inequality (2) holds, from the point of view of the cost of trading, the central bank prefers the common knowledge setup, whereas from the effectiveness' point of view the central bank prefers the secret setup. Hence, the analysis of the losses will depend upon which of the two terms dominates. To this end, we study particular cases according to extreme values for the parameters σ_η^2 , q , σ_ε^2 and σ_f^2 .

First, we analyse the consistency and inconsistency of intervention. We have shown that if the intervention is consistent with fundamentals, secrecy will provide a higher expected

profit, whereas the expected effectiveness of the central bank intervention will be higher under common knowledge. Depending on the values of the remaining parameters, one of these two effects will dominate. In the case of inconsistent intervention, the common knowledge will provide an expected profit, whereas both common knowledge and secrecy are equally and completely ineffective, with this last effect prevailing. We obtain the following corollary regarding the consistency of the intervention.

Corollary 5: *Under a consistent policy ($\sigma_\eta^2 = 0$), the central bank's losses will be lower when its intervention is secret as long as $\frac{\sigma_\varepsilon^2}{q^2\sigma_f^2} > \sqrt{2}-1$. On the other hand, when σ_η^2 is high, that is, under an inconsistent policy, the losses of the central bank under common knowledge and under secrecy are similar.*

The intuition for this result is as follows. Under a consistent policy ($\sigma_\eta^2 = 0$), the expressions for the loss function of the central bank are given by

$$\begin{aligned} L^{CK} &= q(s_0 - e)^2 \text{ and} \\ L^S &= -\bar{\lambda}\sigma_\varepsilon^2 + q\left(\frac{\sigma_f^2}{2(q\bar{\lambda} + 1)} + (s_0 - e)^2\right). \end{aligned}$$

Notice that these expressions show that under secrecy the profitability is higher than under common knowledge, even though the effectiveness is lower. The difference in profits increases the higher is σ_ε^2 . On the other hand, the effectiveness of secrecy with respect to common knowledge becomes similar when σ_f^2 is small, whereas the difference in effectiveness is not relevant for the central bank when q is small. Hence, when the volume of trading is high, the weight given by the central bank to exchange rate stabilization is small and the variance of the fundamental is small, the term that dominates the expected losses is the cost of trading and, consequently, the central bank will prefer to intervene secretly.

Under an inconsistent intervention ($\sigma_\eta^2 \rightarrow \infty$), both expressions for the losses tend to

infinite and, moreover, they are similar. This is due to the fact that the measure of the effectiveness of the central bank converges to zero in a similar way under both secrecy and common knowledge. Note that this result would also occur in Vitale's (1999) model when the variance of the monetary authority's target tends to infinity.

The model presented in this article can also have predictions for intermediate values of σ_η^2 , which will correspond to an intermediate case where intervention will not be completely consistent or inconsistent. The choice of secrecy or common knowledge will then depend on the remaining parameters of the model $(q, \sigma_\varepsilon^2, \sigma_f^2)$. First, we analyze the unconditional expected losses of the central bank in the following two extreme cases: $q = 0$ and $q \rightarrow \infty$. Notice that in the former case the expected losses coincide with the cost of trading. Therefore, the positive relationship between the cost of trading and market liquidity together with Corollary 1 allow us to conclude that the cost of trading is higher in the common knowledge, and therefore, the central bank prefers to conceal the target level when its intervention is purely speculative. On the other hand, when $q \rightarrow \infty$, the central bank is only concerned with the stability of the exchange rate. Consequently, Corollary 4 shows that the central bank is more effective in the secret setup, and since this is its unique objective, it prefers this scenario. These results are summarized in the following corollary:

Corollary 6: *Both when the central bank is purely speculative ($q = 0$) and when the central bank is only concerned with the stability of the exchange rate ($q \rightarrow \infty$), it prefers to conceal its target level whenever $\sigma_\eta^2 > 0$.*

For the extreme values of q , the central bank prefers to conceal its target. However, it is important to point out that for intermediate values of q the central bank may prefer to reveal its target level. For instance, we only have to consider a consistent policy that satisfies $\frac{\sigma_\varepsilon^2}{q^2 \sigma_f^2} < \sqrt{2} - 1$.

Next, we analyze the expected losses of the central bank in the two extreme cases for the volume of noise trading. When $\sigma_\varepsilon^2 \rightarrow 0$, in both markets the cost of trading is null. Thus, the comparison of the losses of the central bank is reduced to the contrast of its effectiveness, which is done in Corollary 4. Therefore, we obtain that the central bank will prefer to intervene secretly whenever $q\sigma_\eta^2 \neq 0$. On the other hand, when $\sigma_\varepsilon^2 \rightarrow \infty$, in both markets the cost of trading converges to $-\infty$, whereas the second term of the loss function of the central bank is bounded. In addition, Corollary 1 implies that the cost of trading is smaller under secrecy and, hence, this setup is preferred. The following corollary sets out these results:

Corollary 7: *When the volume of noise trading is low enough, the central bank's losses will be lower when its intervention is secret whenever $q\sigma_\eta^2 \neq 0$. When the volume of noise trading is high enough, the central bank also prefers to conceal its target level.*

Finally, we compare the expected losses of the central bank in the two extreme cases for the variance of the fundamental. When there is no asymmetric information ($\sigma_f^2 \rightarrow 0$), the central bank is indifferent between the two frameworks compared since, in each of them, market makers perfectly know the fundamental. On the other hand, when there is a lot of asymmetry of information, the more opaque framework allows the monetary authority to better exploit its privileged information. However, in such an uncertain framework, market makers set an exchange rate more distant from the target level under secrecy. Then, the following result shows that the increase in profitability under secrecy does not compensate for the loss of effectiveness and, hence, the central bank prefers to act publicly. These results are summarised in the following corollary:

Corollary 8: *When the variance of the fundamental converges to zero, and thus the central bank does not possess superior information, the central bank is indifferent between*

both scenarios. By contrast, when the variance of the fundamental converges to infinity the central bank prefers the common knowledge setup.

4.4 Predictions of the model and empirical evidence

Some of the results obtained in the preceding analysis are in accordance with empirical evidence. For instance, Dominguez (2003) reports the average timings of intervention of the Fed and the Bundesbank for the period 1987 to 1995. It is interesting to note that the average timing of interventions of the Bundesbank is reported around the lunch hour in Frankfurt, and for the Fed it is around 10 in the morning, times where both the European and American markets are open and the volume of trading should be higher. In our model, this would be equivalent to a higher σ_ε^2 , and according to Corollary 7, the central bank would prefer to act secretly. >From this point of view it is also interesting to consider that currencies like the dollar, the euro and the yen, which are commonly accepted internationally as means of payment, will be naturally subject to higher σ_ε^2 than other currencies that are not as widely accepted. This would be, again, in accordance with the tendency of the Fed and the Bank of Japan to intervene secretly, compared to, for instance, the Swiss National Bank, who intervenes publicly.

Further, Chiu (2003) raises the question of whether economies with smaller and less liquid currency markets can afford to be less visible in their foreign exchange intervention. Chiu acknowledges that there are few studies that have looked at this issue, but provides the example of the Monetary Authority of Singapore, whose interventions are mostly carried out in secrecy. Again, this would be in line with the predictions of the model presented here, as smaller markets will present lower volume of noise trading.

Also, according to our model, when central banks are very concerned about profitability

(q is very small), they will prefer to intervene secretly. In this respect, it is interesting to note that some central banks have insisted that their interventions are profitable (see Bank of England (1983)) or believe that their trading desks could trade profitably (Neely (2006)).

Finally, this article has offered predictions that should be further analysed empirically. For instance, it has been shown that factors such as the volume of noise trading might influence the choice of intervention under inconsistency. In order to study the choices between secret and public central bank intervention, the inclusion of these factors in the empirical analysis could help to improve our understanding of central bank interventions.

5 Conclusion

In this article, we have developed a microstructure model of central bank intervention in the foreign exchange market under the signalling channel that allows for intervention to be consistent or inconsistent with fundamentals. If we accept that the central bank possesses some private information and its objectives take into account profitability and an exchange rate target, we have offered a rationale for the secrecy of interventions. We have illustrated that when the intervention is consistent with fundamentals, the central bank will prefer to keep its target secret in the presence of high volume of noise trading, when it is very concerned about its profitability and when the asymmetry of information is low. Further, the central bank also prefers secrecy when its intervention is purely speculative or purely for stabilization purposes. Similarly, for extreme values of noise trading, secrecy is also preferred. By contrast, when the asymmetry of information between the central bank and the market is significant, it will not be optimal for the central bank to keep its target secret.

We have also demonstrated that there is a trade-off between the cost of trading and effectiveness: the conditions that provide for lower cost of trading under secrecy are asso-

ciated with a higher effectiveness under common knowledge. Further, we have also shown that the profitability of the central bank intervention is inversely related to the liquidity and efficiency of the market.

The analysis developed in this article could be extended to consider, for instance, different structures of agents or different objectives for the central bank. The model considered here assumes that the central bank deals only with market makers (dealers). Nowadays, with electronic trading services, some central banks are also using brokers. An interesting extension would be to include both market makers and brokers in this model.

Further, there is evidence that some central banks traditionally preferred to trade with just a few market makers. In particular, Peiers (1997) found empirical evidence of the Deutsche Bank's price leadership, sometimes up to one hour, prior to the Bundesbank intervention reports. On the other hand, it has been reported (see Dominguez (2003)) that the Fed, for instance, generally attempts to use a wide and variable selection of banks for the intervention transactions in order not to give any of them unfair advantage. Another possible extension would involve investigating whether in these two different structures secrecy would still be an optimal choice, and if so, under what circumstances.

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Appendix

Proof of Proposition 1: First, we consider the central bank's problem. Suppose that the central bank conjectures that the pricing rule is as follows:

$$s_1 = \mu + \lambda(x + \varepsilon). \quad (7)$$

Then, the bank central solves the following optimization problem:

$$\min_x E \left((\mu + \lambda(x + \varepsilon) - f)x + q(\mu + \lambda(x + \varepsilon) - t)^2 \mid f, t \right).$$

The first order condition of this problem is

$$2\lambda x(q\lambda + 1) + (\mu - f + 2q(\mu - t)\lambda) = 0,$$

and its second order condition implies that this function is strictly convex. Operating we obtain

$$x = \beta(f - s_0) + \frac{q}{(q\lambda + 1)}t + \frac{1}{2\lambda(q\lambda + 1)}s_0 - \frac{\mu(2q\lambda + 1)}{2\lambda(q\lambda + 1)}, \quad (8)$$

with

$$\beta = \frac{1}{2\lambda(q\lambda + 1)}. \quad (9)$$

Next, let us focus on the dealers' problem. Recall that competition among market makers implies that

$$s_1 = E(f \mid x + \varepsilon, t).$$

Using (8), $(x + \varepsilon, t)$ is informationally equivalent to $(\beta(f - s_0) + \varepsilon, t)$. Hence,

$$s_1 = E(f \mid \beta(f - s_0) + \varepsilon, t).$$

Using the normality assumption, the previous expression becomes

$$s_1 = s_0 + (A, B) \begin{pmatrix} \beta(f - s_0) + \varepsilon \\ t - e \end{pmatrix} \quad (10)$$

with

$$(A, B) = \text{cov}(f, (\beta(f - s_0) + \varepsilon, t)) \text{var}^{-1}(\beta(f - s_0) + \varepsilon, t). \quad (11)$$

Direct computations yield

$$\text{cov}(f, (\beta(f - s_0) + \varepsilon, t)) = (\beta\sigma_f^2, \sigma_f^2)$$

and

$$\text{var}(\beta(f - s_0) + \varepsilon, t) = \begin{pmatrix} \beta^2\sigma_f^2 + \sigma_\varepsilon^2 & \beta\sigma_f^2 \\ \beta\sigma_f^2 & \sigma_f^2 + \sigma_\eta^2 \end{pmatrix}.$$

Inserting these expressions in (11), we obtain

$$A = \frac{\sigma_\eta^2\sigma_f^2\beta}{\sigma_\varepsilon^2(\sigma_f^2 + \sigma_\eta^2) + \beta^2\sigma_f^2\sigma_\eta^2} \text{ and } B = \frac{\sigma_\varepsilon^2\sigma_f^2}{\sigma_\varepsilon^2(\sigma_f^2 + \sigma_\eta^2) + \beta^2\sigma_f^2\sigma_\eta^2} = A\sigma_\eta^{-2}\beta^{-1}\sigma_\varepsilon^2.$$

Combining (7) and (10), we find

$$\mu = s_0 - A \left(\frac{q}{(q\lambda + 1)}t + \frac{1}{2\lambda(q\lambda + 1)}s_0 - \frac{\mu(2q\lambda + 1)}{2\lambda(q\lambda + 1)} \right) + B(t - e) \quad (12)$$

and

$$\lambda = A. \quad (13)$$

Manipulating these expressions and using (9), it follows that

$$\mu = s_0 + 2\lambda q(s_0 - t) + \frac{4(q\lambda + 1)^2\sigma_\varepsilon^2\lambda^2}{\sigma_\eta^2}(t - e)$$

and λ is the unique positive root of the following equation:¹³

$$4\lambda^2(q\lambda + 1)^2\sigma_\varepsilon^2(\sigma_f^2 + \sigma_\eta^2) = (2q\lambda + 1)\sigma_\eta^2\sigma_f^2. \quad (14)$$

Finally, substituting the expression of μ and the previous equality in (8) and (7), the expressions of x and s_1 given in the statement of this proposition are obtained. ■

¹³Descartes' rule provides the uniqueness of the positive root of this polynomial.

Proof of Proposition 2: Performing the same computations as in the proof of Proposition 1, (8) and (9) are also obtained. Since market makers now do not know t , the pricing rule selected in this framework satisfies:

$$s_1 = E(f|x + \varepsilon).$$

Using (8) and the normality assumption, we find

$$s_1 = s_0 + \lambda \left(\beta(f - s_0) + \frac{q}{q\lambda + 1} (t - e) + \varepsilon \right), \quad (15)$$

with

$$\lambda = \frac{\left(\beta + \frac{q}{q\lambda + 1} \right) \sigma_f^2}{\left(\beta + \frac{q}{q\lambda + 1} \right)^2 \sigma_f^2 + \left(\frac{q}{q\lambda + 1} \right)^2 \sigma_\eta^2 + \sigma_\varepsilon^2}.$$

Plugging the expression of beta (9) into the previous equality and after some computations we obtain that λ is the unique positive root of the following equation:

$$4\lambda^2 q^2 \sigma_\eta^2 + 4\lambda^2 (q\lambda + 1)^2 \sigma_\varepsilon^2 = (2q\lambda + 1) \sigma_f^2. \quad (16)$$

Moreover, combining (7) and (15), using (8), it follows that

$$\mu = s_0 - 2\lambda q (e - s_0).$$

Finally, substituting the expression for μ in (8) and (7), the expressions for x and s_1 given in the statement of this proposition are obtained. ■

Proof of Corollary 1: First, we focus on the comparison of market liquidity. Since λ^{CK} is the unique positive root of $g(\lambda)$ and $\lim_{\lambda \rightarrow \infty} g(\lambda) = \infty$, the inequality $\lambda^S \leq \lambda^{CK}$ is equivalent to $g(\lambda^S) \leq 0$. Developing the left-hand side of this inequality and taking into account that λ^S is the unique positive root of $h(\lambda)$, it follows that the previous inequality implies that

$$\sigma_f \sigma_\varepsilon + q\lambda^S \sigma_f \sigma_\varepsilon - q\sigma_\eta^2 \leq 0. \quad (17)$$

We distinguish two cases: 1) $\sigma_f \sigma_\varepsilon - q \sigma_\eta^2 \geq 0$, and 2) $\sigma_f \sigma_\varepsilon - q \sigma_\eta^2 < 0$. In the former we have that $\lambda^S > \lambda^{CK}$. In the latter, (17) implies that $\lambda^S \leq \frac{q \sigma_\eta^2 - \sigma_f \sigma_\varepsilon}{q \sigma_f \sigma_\varepsilon}$. Since λ^S is the unique positive root of $h(\lambda)$ and $\lim_{\lambda \rightarrow \infty} h(\lambda) = \infty$, the previous inequality is equivalent to $h(\frac{q \sigma_\eta^2 - \sigma_f \sigma_\varepsilon}{q \sigma_f \sigma_\varepsilon}) \geq 0$. Operating we find that, when $\sigma_f \sigma_\varepsilon < q \sigma_\eta^2$, the previous inequality holds if and only if (2) is satisfied. Combining the results derived in the two cases, we obtain the desired result related to market liquidity.

Second, we consider market efficiency. Recall that when the target level is common knowledge, the information set of the market makers is $(x^{CK} + \varepsilon, t)$, which is informationally equivalent to $(\beta^{CK} f + \varepsilon, t)$. Using this fact and the normality assumption, operating, we obtain

$$\text{var}(f|I_m^{CK}) = \frac{\sigma_\varepsilon^2 \sigma_\eta^2 \sigma_f^2}{\left(\sigma_f^2 \sigma_\varepsilon^2 + \sigma_\eta^2 \sigma_\varepsilon^2 + (\beta^{CK})^2 \sigma_f^2 \sigma_\eta^2\right)}.$$

Substituting the expression of β^{CK} given in (9) and using (14), it follows that

$$\text{var}(f|I_m^{CK}) = \frac{(2q\lambda^{CK} + 1) \sigma_\eta^2 \sigma_f^2}{2(q\lambda^{CK} + 1) (\sigma_f^2 + \sigma_\eta^2)}.$$

When the target level is secret, market makers only observe $x^S + \varepsilon$, which is informationally equivalent to $\beta^S f + \frac{q}{(q\lambda^S + 1)} t + \varepsilon$. Using this fact and the normality assumption, operating, it follows that

$$\text{var}(f|I_m^S) = \frac{\sigma_f^2 \left(\left(\frac{q}{q\lambda^S + 1} \right)^2 \sigma_\eta^2 + \sigma_\varepsilon^2 \right)}{\left(\beta^S + \frac{q}{q\lambda^S + 1} \right)^2 \sigma_f^2 + \left(\frac{q}{q\lambda^S + 1} \right)^2 \sigma_\eta^2 + \sigma_\varepsilon^2}.$$

Using the expression of β^S given in (9) and (16), we find

$$\text{var}(f|I_m^S) = \frac{\sigma_f^2}{2(q\lambda^S + 1)}.$$

Hence, the difference between the market efficiency in both setups is given by:

$$\text{var}^{-1}(f|I_m^{CK}) - \text{var}^{-1}(f|I_m^S) = \frac{2 \left((q\lambda^{CK} + 1) 2q \left(\frac{\sigma_f^2}{2q\sigma_\eta^2} - \lambda^S \right) + q \left(\lambda^S - \lambda^{CK} \right) \right)}{\sigma_f^2 (2q\lambda^{CK} + 1)}. \quad (18)$$

In addition, since λ^S is the unique positive root of $h(\lambda)$ and $\lim_{\lambda \rightarrow \infty} h(\lambda) = \infty$, $\lambda^S \geq \frac{\sigma_f^2}{2q\sigma_\eta^2}$ is equivalent to $h\left(\frac{\sigma_f^2}{2q\sigma_\eta^2}\right) \leq 0$. Direct computations imply that this inequality holds if and only if (2) is satisfied, which is equivalent to $\lambda^S \leq \lambda^{CK}$, as it has been derived in the first part of this proof. Thus, we have that $\lambda^S \geq \frac{\sigma_f^2}{2q\sigma_\eta^2}$ if and only if $\lambda^S \leq \lambda^{CK}$. Combining this result with (18), we can conclude that $\text{var}^{-1}(f|I_m^{CK}) \leq \text{var}^{-1}(f|I_m^S)$ if and only if (2). ■

Proof of Corollary 2: Since all the proofs are very similar, we have only included the one corresponding to λ^{CK} increases in σ_η^2 . Let us define the following function:

$$G(\lambda, \sigma_\eta^2) = 4\lambda^2 (q\lambda + 1)^2 \sigma_\varepsilon^2 (\sigma_f^2 + \sigma_\eta^2) - (2q\lambda + 1) \sigma_f^2 \sigma_\eta^2.$$

By Proposition 1, we know that $G(\lambda^{CK}, \sigma_\eta^2) = 0$. Applying the Implicit Function Theorem, we know that $\frac{d\lambda^{CK}}{d\sigma_\eta^2} = -\frac{\frac{\partial G(\lambda^{CK}, \sigma_\eta^2)}{\partial \sigma_\eta^2}}{\frac{\partial G(\lambda^{CK}, \sigma_\eta^2)}{\partial \lambda}}$. Operating we find

$$\frac{d\lambda^{CK}}{d\sigma_\eta^2} = -\frac{4\left(\lambda^{CK}\right)^2 \left(q\lambda^{CK} + 1\right)^2 \sigma_\varepsilon^2 - \left(2q\lambda^{CK} + 1\right) \sigma_f^2}{8\lambda^{CK} \left(2q\lambda^{CK} + 1\right) \left(q\lambda^{CK} + 1\right) \sigma_\varepsilon^2 \left(\sigma_f^2 + \sigma_\eta^2\right) - 2q\sigma_f^2 \sigma_\eta^2}.$$

Taking into account that $G(\lambda^{CK}, \sigma_\eta^2) = 0$, the numerator of the fraction on the right hand side is negative and the denominator is positive. Hence, $\frac{d\lambda^{CK}}{d\sigma_\eta^2} \geq 0$. On the other hand, the extreme values of λ^{CK} follow from direct computations. ■

Proof of Corollary 3: This proof is omitted since it is very similar to the proof of Corollary 2. ■

Proof of Corollary 4: Consider the case in which the target level of the central bank is common knowledge. From Proposition 1, it follows that

$$\begin{aligned} s_1^{CK} - t &= \frac{4\sigma_\varepsilon^2 (\lambda^{CK})^2 \left(q\lambda^{CK} + 1\right)^2 - \sigma_\eta^2 \left(2q\lambda^{CK} + 1\right)}{2\left(q\lambda^{CK} + 1\right) \sigma_\eta^2} (f - s_0) + \\ &+ \frac{2\sigma_\varepsilon^2 (\lambda^{CK})^2 \left(q\lambda^{CK} + 1\right) - \sigma_\eta^2}{\sigma_\eta^2} (\eta - e + s_0) + \lambda^{CK} \varepsilon - e + s_0. \end{aligned}$$

>From this expression and performing some computations, using (14), we obtain (5). The computations related to (6) have been omitted, since they are very similar to the common knowledge case.

Further, from (5) and (6), we have that $E\left((s_1^{CK} - t)^2\right) \geq E\left((s_1^S - t)^2\right)$ holds if and only if $\lambda^S \geq \frac{\sigma_f^2(\sigma_f^2 + 2\sigma_\eta^2) + \sigma_f^2 q(\sigma_f^2 + 3\sigma_\eta^2)\lambda^{CK}}{\sigma_\eta^2 q(3\sigma_f^2 + 4\sigma_\eta^2 + 2q\lambda^{CK}(\sigma_f^2 + 2\sigma_\eta^2))}$ is satisfied, which is equivalent to showing $h\left(\frac{\sigma_f^2(\sigma_f^2 + 2\sigma_\eta^2) + \sigma_f^2 q(\sigma_f^2 + 3\sigma_\eta^2)\lambda^{CK}}{\sigma_\eta^2 q(3\sigma_f^2 + 4\sigma_\eta^2 + 2q\lambda^{CK}(\sigma_f^2 + 2\sigma_\eta^2))}\right) \leq 0$. Developing the left-hand side of this inequality, using (14), it follows that

$$\text{sign}\left(h\left(\frac{\sigma_f^2(\sigma_f^2 + 2\sigma_\eta^2) + \sigma_f^2 q(\sigma_f^2 + 3\sigma_\eta^2)\lambda^{CK}}{\sigma_\eta^2 q(3\sigma_f^2 + 4\sigma_\eta^2 + 2q\lambda^{CK}(\sigma_f^2 + 2\sigma_\eta^2))}\right)\right) = \text{sign}\left(\sigma_f^2 - 2\lambda^{CK} q\sigma_\eta^2\right).$$

Since λ^{CK} is decreasing in σ_ε^2 and $\lambda^{CK} = \frac{\sigma_f^2}{2q\sigma_\eta^2}$ when $\sigma_\varepsilon = \frac{2\sigma_\eta^4 q}{\sigma_f(\sigma_f^2 + 2\sigma_\eta^2)}$, it follows that the last inequality is equivalent to (2). ■

Proof of Corollary 5: We consider first the case where $\sigma_\eta^2 = 0$. In this case, $L^{CK} = q(s_0 - e)^2$ and $L^S = \frac{(2q^2\bar{\lambda}^2 - 1)\sigma_f^2}{4(q\bar{\lambda} + 1)^2\bar{\lambda}} + q(s_0 - e)^2$ because of (3). Then, $L^S < L^{CK}$ can be rewritten as $\bar{\lambda} < \frac{1}{\sqrt{2q}}$, which is equivalent to $\frac{\sigma_\varepsilon^2}{q^2\sigma_f^2} > \sqrt{2} - 1$, since $\bar{\lambda}$ is the unique positive root of (3).

Next, we consider the case where $\sigma_\eta^2 \rightarrow \infty$. Here, $\lim_{\sigma_\eta^2 \rightarrow \infty} \frac{L^{CK}}{L^S} = \lim_{\sigma_\eta^2 \rightarrow \infty} \frac{E\left((s_1^{CK} - t)^2\right)}{E\left((s_1^S - t)^2\right)} = 1$. The first equality holds because the cost of trading is finite and the second term of the loss function is infinite in both setups. The last equality is derived from some direct computations. ■

Proof of Corollary 7: When $\sigma_\varepsilon^2 \rightarrow 0$, from Corollaries 2 and 3, we have that $\lambda^{CK} \rightarrow \infty$ and $\lambda^S \rightarrow \frac{\sigma_f^2 + \sqrt{\sigma_f^4 + 4\sigma_f^2\sigma_\eta^2}}{4q\sigma_\eta^2}$. Moreover, using from (14) and (16), it follows that the cost of trading is null in both scenarios. Therefore, the comparison of the losses of the central bank is reduced to the contrast of its effectiveness. >From Corollary 4, it follows that $L^{CK} > L^S$ whenever $q\sigma_\eta^2 \neq 0$.

On the other hand, when $\sigma_\varepsilon^2 \rightarrow \infty$, we find that

$$\lim_{\sigma_\varepsilon^2 \rightarrow \infty} \frac{L^{CK}}{L^S} = \lim_{\sigma_\varepsilon^2 \rightarrow \infty} \frac{-\lambda^{CK} \sigma_\varepsilon^2 + q \left(\frac{(\sigma_f^2 + 2(q\lambda^{CK} + 1)\sigma_\eta^2)\sigma_\eta^2}{2(q\lambda^{CK} + 1)(\sigma_f^2 + \sigma_\eta^2)} + (s_0 - e)^2 \right)}{-\lambda^S \sigma_\varepsilon^2 + q \left(\frac{\sigma_f^2 - 2(q\lambda^S - 1)\sigma_\eta^2}{2(q\lambda^S + 1)} + (s_0 - e)^2 \right)} = \lim_{\sigma_\varepsilon^2 \rightarrow \infty} \frac{\lambda^{CK}}{\lambda^S}.$$

Notice that from (14) and (16),

$$\lim_{\sigma_\varepsilon^2 \rightarrow \infty} \left(\frac{\lambda^{CK}}{\lambda^S} \right)^2 = \lim_{\sigma_\varepsilon^2 \rightarrow \infty} \frac{\frac{(2q\lambda^{CK} + 1)\sigma_\eta^2\sigma_f^2}{4(q\lambda^{CK} + 1)^2\sigma_\varepsilon^2(\sigma_f^2 + \sigma_\eta^2)}}{\frac{(2q\lambda^S + 1)\sigma_f^2}{4(q^2\sigma_\eta^2 + (q\lambda^S + 1)^2\sigma_\varepsilon^2)}} = \frac{\sigma_\eta^2}{\sigma_f^2 + \sigma_\eta^2} < 1.$$

Therefore, combining these limits we obtain that $\lim_{\sigma_\varepsilon^2 \rightarrow \infty} \frac{L^{CK}}{L^S} < 1$. Using the fact that when σ_ε^2 is high enough L^S is negative, the last limit implies that in this case $L^{CK} > L^S$. ■

Proof of Corollary 8: When $\sigma_f^2 \rightarrow 0$, from Corollaries 2 and 3, we have that $\lambda^{CK} \rightarrow 0$ and $\lambda^S \rightarrow 0$. Direct computations imply that in that case $L^{CK} = L^S = q(\sigma_\eta^2 + s_0 - e)^2$, and therefore, the central bank is indifferent between the two setups.

Consider now the case where $\sigma_f^2 \rightarrow \infty$. Using Corollary 2, we have that the $\lim_{\sigma_f^2 \rightarrow \infty} L^{CK}$ is finite. On the other hand, using (16), we can rewrite the expression for the loss function as $L^S = \frac{(q\sigma_\eta^2 + \sigma_\varepsilon^2\lambda^S(2q^2(\lambda^S)^2 - 1))}{(2q\lambda^S + 1)} + q(s_0 - e)^2$, which converges to infinity when $\sigma_f^2 \rightarrow \infty$ because of Corollary 3. Therefore, the central bank prefers the common knowledge setup. ■

References

- [1] Bank of England, 1983. Intervention, stabilisation and profits. Quarterly Bulletin 23, 384 – 391.
- [2] Barnett, R., Ozerturk, S., 2007. The advantage of showing your hand selectively in foreign exchange interventions. European Journal of Political Economy 23, 228-244.

- [3] Beine, M. and O. Bernal, 2007. Why do central banks intervene secretly? Preliminary evidence from the BoJ, *Journal of International Financial Markets, Institutions and Money*, forthcoming.
- [4] Beine, M., O. Bernal, J. Gnabo and C. Lecourt, 2007. Intervention policy of the BoJ: a unified approach, mimeo.
- [5] Bhattacharya, U., Weller, P., 1997. The advantage to hiding one's hand: Speculation and central bank intervention in the foreign exchange market. *Journal of Monetary Economics* 39, 251 – 277.
- [6] Chang, Y., Taylor, S., 1998. Intraday effects of foreign exchange intervention by the Bank of Japan. *Journal of International Money and Finance* 17, 191 – 210.
- [7] Chiu, P., 2003. Transparency versus constructive ambiguity in foreign exchange intervention. BIS Working Papers, no. 144.
- [8] Dominguez, K., 2003. The market microstructure of central bank intervention. *Journal of International Economics* 59, 25 – 45.
- [9] Dominguez, K., Frankel, A., 1993. Does Foreign Exchange Intervention Work? Washington D.C.: Institute for International Economics.
- [10] Engel, C. and K. West, 2005. Exchange Rates and Fundamentals. *Journal of Political Economy* 113, 485-517.
- [11] Frenkel, M., C. Pierdzioch and G. Stadtmann (2003), Modeling coordinated foreign exchange market interventions: the case of the Japanese and U.S. interventions in the 1990s, *Review of World Economics* 139, 709-729.

- [12] Humpage, O., 1994. Institutional aspects of U.S. intervention. Federal Reserve bank of Cleveland Economic Review 30, 2 – 19.
- [13] Kyle, A. S., 1985. Continuous auctions and insider trading. Econometrica 53, 1315 – 1335.
- [14] Leahy, M., 1995. The profitability of US intervention in the foreign exchange markets. Journal of International Money and Finance 14, 823 – 844.
- [15] Lyons, R., 2001. The Microstructure Approach to Exchange Rates. MIT Press, Cambridge, MA.
- [16] Manzano, C., 2002. The effect of the transparency of order flows in a dealer market with several securities. Journal of Financial Intermediation 11, 212 – 227.
- [17] Mussa, M., 1981. The Role of Official Intervention. Group of Thirty, New York, NY.
- [18] Neely, C., 2001. The Practice of Central Bank Intervention: Looking Under the Hood. Federal Reserve Bank of St. Louis, May/June.
- [19] Neely, C, 2006. Central Bank Authorities' Beliefs about Foreign Exchange Intervention. Federal Reserve Bank of St. Louis Working Paper Series 2006-045C.
- [20] Peiers, B., 1997. Informed traders, intervention, and price leadership: A deeper view of the microstructure of the foreign exchange market. The Journal of Finance 52, 1589 – 1614.
- [21] Sarno, L. and M. Taylor, 2001. Official intervention in the foreign exchange market: is it effective and, if so, how does it work?, Journal of Economic Literature 39, 839-868.
- [22] Saacke, P., 2002. Technical analysis and the effectiveness of central bank intervention. Journal of International Money and Finance 21, 459 – 479.

- [23] Sjö, B., Sweeney, R., 2001. The foreign exchange costs of central bank intervention: Evidence from Sweden. *Journal of International Money and Finance* 20, 219 – 247.
- [24] Sweeney, R., 2000. Does the Fed beat the foreign exchange market? *Journal of Banking and Finance* 24, 665-694.
- [25] Vitale, P., 1999. Sterilised central bank intervention in the foreign exchange market. *Journal of International Economics* 49, 245 – 267.