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to estimate the quantile**

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# Testing extreme value copulas to estimate the quantile

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## Abstract

Testing whether or not data belongs could be generated by a family of extreme value copulas is difficult. We generalize a test and we prove that it can be applied whatever the alternative hypothesis. We also study the effect of using different extreme value copulas in the context of risk estimation. To measure the risk we use a quantile. Our results have motivated by a bivariate sample of losses from a real database of auto insurance claims. Methods are implemented in R.

**Keywords:** Extreme value copula, Extreme value distributions, Quantile

## 1 Introduction

Let  $S$  be the sum of  $k$  dependent random variables  $(X_1, \dots, X_k)'$ , i.e.  $S = X_1 + \dots + X_k$ . The distribution of  $S$  depends on the multivariate distribution, i.e. the relationship between random variables  $X_j$ ,  $j = 1, \dots, k$  (see [33] for a review of the construction methods of multivariate distributions). Analyzing the distribution of  $S$  is essential in finance and insurance for quantifying

the risk of loss. In this regard, there are studies that have analyzed the stochastic behavior of the sum of dependent risks and the way in which the dependency between these marginal risks may affect the total risk of loss (see [12], [25], [11], [9] and [21]). The aim of this paper is to analyze the test proposed by Kojadinovic et al. in [26] that allows to test whether or not our data have been generated by an extreme value copula. We conclude that weak convergence of the test statistic is true for any of the alternative hypothesis. Using a real data base, we have analyzed how the error in the selection of the copula can affect the risk estimate. Throughout this paper we simplify the notation for the bivariate case.

As noted by Fisher in [14], copulas are interesting for statisticians due to two basic reasons: firstly, because of their application in the study of nonparametric measures of dependence and, secondly, as a starting point for constructing multivariate distributions representing dependency structures, even when the marginals follow extreme value distributions (EVD). Also, we know that the choice of the marginals may be crucial to model the dependency behavior of variables. According to Nelsen (see [27]), in the coupling of the joint distribution with marginals, the copula captures the link between the two behaviors. The relationship between the joint distribution and the marginals is established in the fundamental theorem proposed by Sklar in [32]. This theorem shows that a bivariate cumulative distribution function (CDF)  $H$  of a random vector of variables  $(X_1, X_2)$  with marginal cumulative distribution functions (CDFs)  $F_1$  and  $F_2$  includes a copula  $C$  according to the following expression:

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \forall x_1, x_2 \in \mathbb{R}. \quad (1)$$

Due to the fact that the joint distribution (and therefore the dependency structure) is unknown, specific tests for choosing the best copula are necessary. This has been the motivation for developing tests for the adequacy of copulas. It is worth mentioning the paper by Genest and Rivest (see [15]) on inference for bivariate Archimedean copulas, the test proposed in [31] on the positive quadrant dependence hypothesis and, finally, the test of symmetry in bivariate

copulas introduced in [29].

Regarding the inference for extreme value copulas, we can mention the test proposed in [18] based on a Cramér-von Mises statistic and the test analyzed in [19] based on an  $U$ -statistic. However, Kojadinovic et al. in [26] uses the *max – stable* property to test the adequacy of an extreme value copula that is also based on the Cramér-von Mises statistic. In our study we find a similar result for the bivariate case and we obtain the weak convergence of the statistic proposed in the general case.

In section 2, first, we present our main result and, second, we describe three examples of copulas which are extreme value copulas: Gumbel, Galambos and Hustler-Reiss. In section 3 we describe a real database of auto insurance claims which we use in the empirical application. In section 4 we report the results of our empirical study, firstly we apply the test described in section 2 and, secondly, calculating the quantile using different extreme value copulas and comparing these results with those obtained when using a widely known non extreme value copula, such as a Gaussian copula. We use two alternative marginal distributions and we compare them: the log-normal, that is a EVD Type I (Gumbel), and the Champernowne distribution, which converges to a Pareto in the tail and therefore is an EVD Type II (Frechet). We also remark that the Champernowne distribution looks more like a log-normal near 0. We conclude in section 5.

## 2 Test for extreme value copulas

A copula is *max – stable* if for every positive real number  $r$  and all  $u_1, u_2$  in  $[0, 1]$ ,  $C(u_1, u_2) = C^r(u_1^{1/r}, u_2^{1/r})$ . Then we formulate the null hypothesis and its alternative as:

$$\begin{cases} H_0^r : C(u_1, u_2) = C^r(u_1^{1/r}, u_2^{1/r}), & \forall u_1, u_2 \in [0, 1], \forall r > 0 \\ H_1^r : C(u_1, u_2) \neq C^r(u_1^{1/r}, u_2^{1/r}), & \exists u_1, u_2 \in [0, 1], \exists r > 0 \end{cases}.$$

Specifically we need to test the *max – stable* hypothesis,

$$\begin{cases} H_0 : \bigcap_{r>0} H_0^r \\ H_1 : \bigcup_{r>0} H_1^r, \end{cases}$$

in practice we only can test  $H_0^r$  for some values of  $r$ .

Let  $(X_{i1}, X_{i2}), \forall i = 1, \dots, n$  be a bivariate sample of  $n$  independent and identically distributed observations. We consider the functions:

$$\begin{aligned} \mathbb{D}_n^r(u_1, u_2) &= \sqrt{n} \left( C_n(u_1, u_2) - C_n^r(u_1^{1/r}, u_2^{1/r}) \right) \\ \mathbb{D}^r(u_1, u_2) &= \sqrt{n} \left( C(u_1, u_2) - C^r(u_1^{1/r}, u_2^{1/r}) \right), \end{aligned}$$

where  $C_n(u_1, u_2)$  is the empirical copula defined as:

$$C_n(u_1, u_2) = \frac{1}{n} \sum_{i=1}^n \mathbf{I}(F_{1n}(X_{i1}) \leq u_1, F_{2n}(X_{i2}) \leq u_2), \quad u_1, u_2 \in [0, 1]^2, \quad (2)$$

where  $F_{1n}$  and  $F_{2n}$  are empirical marginal distributions. To test the *max – stable* property we need to analyze if we can use  $\mathbb{D}_n^r(u_1, u_2)$  as an estimator of  $\mathbb{D}^r(u_1, u_2)$ . Then we find the convergence to a Gaussian process of the difference  $\mathbb{D}_n^r(u_1, u_2) - \mathbb{D}^r(u_1, u_2)$ .

We use the result by Fermanian et al. in [13] for the weak convergence of the empirical copula  $C_n$  to a Gaussian process  $\mathbb{G}$  in the space of all bounded real-valued functions on  $[0, 1]^2$ , i.e.  $l^\infty([0, 1]^2)$ , which is expressed as follows:

$$\sqrt{n} (C_n(u_1, u_2) - C(u_1, u_2)) \rightsquigarrow \mathbb{G}(u_1, u_2) \quad (3)$$

$$= \mathbb{B}(u_1, u_2) - \partial_1 C(u_1, u_2) \mathbb{B}(u_1, 1) - \partial_2 C(u_1, u_2) \mathbb{B}(1, u_2), \quad (4)$$

where  $\rightsquigarrow$  indicates weak convergence and  $\mathbb{B}$  is a Brownian bridge on  $[0, 1]^2$  with covariance functions:

$$E[\mathbb{B}(u_1, u_2) \mathbb{B}(u'_1, u'_2)] = C(u_1 \wedge u'_1, u_2 \wedge u'_2) - C(u_1, u_2) C(u'_1, u'_2),$$

where  $\wedge$  is the minimum.

**Proposition 1** *If the partial derivatives of a copula  $C(u_1, u_2)$  are continuous then for any  $r > 0$  we have:*

$$\mathbb{D}_n^r(u_1, u_2) - \mathbb{D}^r(u_1, u_2) \rightsquigarrow \mathbb{C}^r(u_1, u_2) = \mathbb{G}(u_1, u_2) - rC^{r-1}(u_1^{1/r}, u_2^{1/r})\mathbb{G}(u_1^{1/r}, u_2^{1/r}), \quad (5)$$

in  $l^\infty([0, 1]^2)$ . Result in (5) is true under  $H_0^r$  and  $H_1^r$ .

Kojadinovic et al. (see [26]) proved the weak convergence under  $H_0^r$  of  $\mathbb{D}_n^r(u_1, u_2)$  towards the same process defined in Proposition 1 but with opposite sign. We have proved weak convergence of the difference  $\mathbb{D}_n^r(u_1, u_2) - \mathbb{D}^r(u_1, u_2)$  that is true under  $H_0^r$  and  $H_1^r$ .

**Proof 1** *In order to prove the result in Proposition 1 we consider the function:*

$$\Gamma : C(u_1, u_2) \longrightarrow C^r(u_1^{1/r}, u_2^{1/r}), r > 0.$$

$\Gamma$  is a differentiable function as proposed by Hadamard (see [30]). We use the Delta functional method to analyze the weak convergence of  $\Gamma(C(u_1, u_2)) = C^r(u_1^{1/r}, u_2^{1/r})$ . To find the derivative of  $C^r(u_1^{1/r}, u_2^{1/r})$  we consider the function:

$$\begin{aligned} h(t) &= \Gamma\left((C + t\Delta)(u_1^{1/r}, u_2^{1/r})\right) - \Gamma\left(C(u_1^{1/r}, u_2^{1/r})\right) \\ &= (C + t\Delta)^r(u_1^{1/r}, u_2^{1/r}) - C^r(u_1^{1/r}, u_2^{1/r}), \end{aligned}$$

where  $t\Delta$  is a function representing a difference. Then we calculate  $\Gamma'$  as the derivative of function  $h$  at  $t = 0$ . Using the expression of the Pascal triangle:

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k,$$

we obtain that:

$$\begin{aligned}
h(t) &= \sum_{k=0}^r \binom{r}{k} C^{r-k}(u_1^{1/r}, u_2^{1/r}) t^k \Delta^k(u_1^{1/r}, u_2^{1/r}) - C^r(u_1^{1/r}, u_2^{1/r}) \\
&= \binom{r}{0} C^r(u_1^{1/r}, u_2^{1/r}) + \binom{r}{1} C^r(u_1^{1/r}, u_2^{1/r}) t \Delta(u_1^{1/r}, u_2^{1/r}) \\
&\quad + \sum_{k=2}^r \binom{r}{k} C^{r-k}(u_1^{1/r}, u_2^{1/r}) t^k \Delta^k(u_1^{1/r}, u_2^{1/r}) - C^r(u_1^{1/r}, u_2^{1/r})
\end{aligned}$$

If we differentiate at  $t = 0$ , we obtain:

$$\frac{\partial h(t)}{\partial t} \Big|_{t=0} = \Gamma'(\Delta) = r C^{r-1}(u_1^{1/r}, u_2^{1/r}) \Delta(u_1^{1/r}, u_2^{1/r}),$$

where  $\Gamma'(\Delta)$  is the first derivative of function  $\Gamma(C(u_1, u_2)) = C^r(u_1^{1/r}, u_2^{1/r})$  with respect to function  $t$  evaluated at  $t = 0$ . The result in Proposition 1 is archived observing that:

$$\mathbb{D}_n^r(u, v) - \mathbb{D}^r(u, v) = \sqrt{n} \left( (C_n(u_1, u_2) - C(u_1, u_2)) - (C_n^r(u_1^{1/r}, u_2^{1/r}) - C^r(u_1^{1/r}, u_2^{1/r})) \right),$$

using the convergence of the empirical copula given by Fermanian et al. (see [13]) we obtain:

$$\sqrt{n} (C_n(u_1, u_2) - C(u_1, u_2)) \rightsquigarrow \mathbb{G}(u_1, u_2),$$

and, finally, applying the Delta functional method, we obtain:

$$\sqrt{n} \left( C_n^r(u_1^{1/r}, u_2^{1/r}) - C^r(u_1^{1/r}, u_2^{1/r}) \right) \rightsquigarrow \Gamma'(\mathbb{G}(u_1, u_2))$$

Under the hypothesis  $H_0$  we have that  $\mathbb{D}^r(u_1, u_2) = 0$  and in this case  $\mathbb{D}_n^r(u_1, u_2)$  weakly converges to process (5).

For hypothesis testing given a fixed  $r$ , we use a Cramér-von Mises statistic:

$$S_n^r = \int_0^1 \int_0^1 (\mathbb{D}_n^r(u_1, u_2))^2 du_1 du_2, \tag{6}$$

As propose by Kojadinovic et al. in [26] for a range of values of  $r, r_1, \dots, r_p$ , the following statistic can be considered:

$$S_n^{r_1, \dots, r_p} = \sum_{i=1}^p S_n^{r_i}. \quad (7)$$

To calculate the critical values we use the method proposed by Van der Vaart in [34], consisting on generating independent copies of  $S_n^r$ . The procedure is as follows:

1. Generating  $N$  independent copies of  $\mathbb{D}_n^r, \mathbb{D}_n^{r,(1)}, \dots, \mathbb{D}_n^{r,(N)}$ , such that

$$(\mathbb{D}_n^r, \mathbb{D}_n^{r,(1)}, \dots, \mathbb{D}_n^{r,(N)}) \rightsquigarrow (\mathbb{D}^r, \mathbb{D}^{r,(1)}, \dots, \mathbb{D}^{r,(N)}),$$

where  $\mathbb{D}^{r,(1)}, \dots, \mathbb{D}^{r,(N)}$  are independent copies of  $\mathbb{D}^r$ .

2. Calculating  $(S_n^{r,(1)}, S_n^{r,(2)}, \dots, S_n^{r,(N)})$  such that:

$$(S_n^r, S_n^{r,(1)}, S_n^{r,(2)}, \dots, S_n^{r,(N)}) \rightsquigarrow (S^r, S^{r,(1)}, S^{r,(2)}, \dots, S^{r,(N)}),$$

where  $(S^{r,(1)}, S^{r,(2)}, \dots, S^{r,(N)})$  are independent copies of  $S^r$ .

3. Obtaining the p-value as:

$$\frac{1}{N} \sum_{k=1}^N \mathbf{I}(S_n^{r,(k)} \geq S_n^r).$$

The Van der Vaart method is implemented in the software R with the function `evTestC()`.

## 2.1 Three examples of extreme value copulas

In the application presented in the next section, we compare three examples of extreme value copulas: Gumbel, Galambos and Hustler-Reiss, which are described in this section.

The functional form of Gumbel copula (see [23]) is given by:

$$C_\theta(u_1, u_2) = \exp\left(-\left[(-\ln(u_1))^\theta + (-\ln(u_2))^\theta\right]^{1/\theta}\right),$$



where  $\theta \in [1, +\infty)$  is the parameter controlling the dependency structure. Note that, the dependence is perfect when  $\theta \rightarrow \infty$ , while independence corresponds to the case when  $\theta = 1$ . For the Gumbel copula, it is well known that lower tail dependence is  $\lambda_L = 0$  and upper tail dependence is  $\lambda_U = 2 - 2^{\frac{1}{\theta}}$ , i.e. the Gumbel copula has upper tail dependence.

The Galambos copula was proposed by Galambos [16] and has the following form:

$$C(u_1, u_2) = u_1 u_2 \exp \left( \left[ (-\ln(u_1))^{-\theta} + (-\ln(u_2))^{-\theta} \right]^{-1/\theta} \right),$$

where the range of  $\theta$  is  $[0, \infty)$  and the upper tail dependence is  $\lambda_U = 2 - 2^{\frac{1}{\theta}}$ .

Another example of extreme value copulas is the Hüstler-Reiss copula that was developed by Hüstler and Reiss in [24]. Its functional form is given by:

$$C(u_1, u_2) = \exp \left( -\hat{u}_1 \Phi \left[ \frac{1}{\theta} + \frac{1}{2} \theta \ln \left( \frac{\hat{u}_1}{\hat{u}_2} \right) \right] - \hat{u}_2 \Phi \left[ \frac{1}{\theta} + \frac{1}{2} \theta \ln \left( \frac{\hat{u}_2}{\hat{u}_1} \right) \right] \right),$$

where the range of  $\theta$  is  $[0, \infty)$  and  $\Phi$  is cdf of the standard Gaussian,  $\hat{u}_1 = -\ln(u_1)$  and  $\hat{u}_2 = -\ln(u_2)$ .

### 3 The data

Our example is motivated by a problem in the context of insurance. When there is an accident, the total cost to be paid to a policyholder is the sum of two components: (1) the material damage and (2) the bodily injury compensation. The insurance company is interested in evaluating the risk of a given claim exceeding a certain amount. So the right-tail quantiles are important to understand the risk that an accident claim is very costly.

We work with a random sample of 518 observations containing two types of costs: **Cost1**, representing property damages and compensation of the loss, and **Cost2**, which corresponds to the expenses related to medical care and hospitalization. In general, cost of bodily injuries is covered by the National Institute of Health, however the insured has to bear the cost of some

medical expenses and rehabilitation, technical assistance, drugs, etc . . . , including compensation for pain, suffering and loss of income.

Bodily injury claims typically take years to be settled. Nevertheless, all the claims in our sample were already settled in 2002, according to the company, (see [9]). Finally, we should mention that the compensation may include payments to third parties that have been damaged in one way or another.

In Table 1 we summarize the descriptive statistics of the sample for **Cost1**, **Cost2** and the Total Cost. The variables **Cost1** and **Cost2** are always positive, and there is a big difference between the corresponding maximum and minimum values. Furthermore, we observe that variables described in Table 1 have right skewness. In Figure 1 we show the histograms where we represent the shape of the distribution associated with the variables **Cost1** and **Cost2**.

Cost	Average	Std.Dev.	Skewness	Min	Max	Median
<b>Cost1</b>	182.80	686.80	15.65	13.00	137900.00	677.00
<b>Cost2</b>	283.92	863.17	8.04	1.00	11855.00	88.00
<b>Total Cost</b>	211.20	752.00	15.27	32.00	149800.00	789.00

Table 1: Descriptive statistics.

The K-Plot (related to Kendall Plot, see [17]) is a visual method that allows us to analyze in a descriptive way if our bivariate data have been generated by an extreme value copula. In Figure 2 we show the K-Plot, that compare the order in real data ( $H$ , pseudo-observations generated by the bivariate empirical distribution) with the order supposing that the data have been generated by the independence copula ( $W$ , expected pseudo-observations). We note that costs have a positive association (as shown in the values of the K-plot above the diagonal, which indicates independence). Almost all points are between the straight line and the boundary curve indicating perfect positive dependence. It seems that according to the increasing values of  $W$ ,

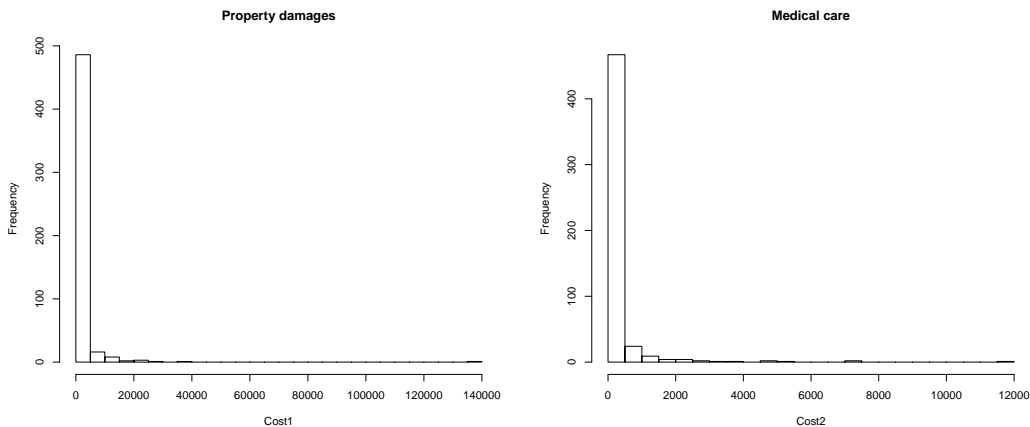


Figure 1: Histograms.

the data is closed to the case of a perfect positive dependence. This means that the higher the severity of the claim, the higher is the correlation between the medical costs and compensation.

## 4 Results

In this section we report the results that we have obtained in an empirical application of the methodology that we have presented. For estimating the total risk of loss, our goal is to determine the dependency structure between the data corresponding to a sample of claims provided by a major insurance company which operates in Spain. For testing if our data are generated by an extreme value copula we calculate the value of the Cramér-Von Mises statistic in (7), with  $r = 3, 4, 5$ . We have estimated the significance level of the test statistic using the method proposed by Van der Vaart in [34]. In total, we generated 1000 independent copies of  $S_n^{3,4,5}$ . The results are shown in Table 2 and allow us to conclude that the analyzed bivariate data are generated by an extreme value copula.

We estimate the parameters of the tree extreme value copulas described in section 2.1:

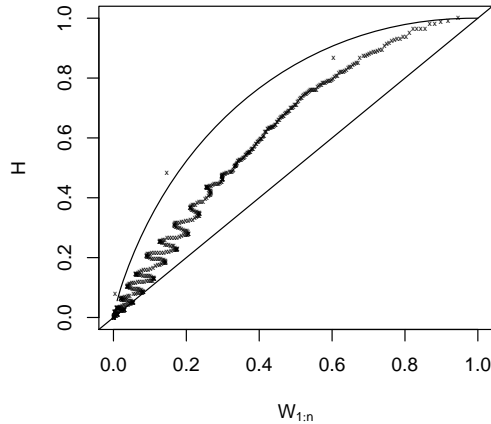


Figure 2: K-Plot associated to copula of (**Cost1**, **Cost2**).

Statistic	Estimation	p-value
$S_n^{3,4,5}$	0.2680	0.1773

Table 2: Cramér-Von Mises statistics.

Gumbel, Galambos and Hüsler-Reiss. In Table 3 we show the estimated parameters for these three copulas together with those obtained for the Gaussian and the t-Student copulas. To estimate the dependence parameter of Gaussian, Gumbel, Galambos and Hüsler-Reiss copulas we have used the inversion of Kendall’s tau method (Itau). To estimate dependence parameter and degree of freedom of t-Student copula we have used maximum likelihood estimation (MLE). For selecting the copula we have used two known statistical information criteria, the Akaike Information Criteria  $AIC = -2 \log L(\theta, u_1, u_2) + 2k$  and the Bayesian Information Criteria  $BIC = -2 \log L(\theta, u, v) + \log(n)k$ , where  $k$  is the number of parameters to be estimated and  $L$  the value of the likelihood function. Also, we have also used the copula information criteria

*CIC* propose by Gronneberg and Hjort [20]. The corresponding results are presented in Table 3, in practice we observe that AIC and CIC values are very similar. We observe that the Gumbel copula is the one that best reflects the dependence structure of our data.

	Gaussian	t-Student	Gumbel	Galambos	Hustler-Reiss
Parameters	0.6193	0.5981 d.f.= 9.6442	1.7397	1.0208	1.4946
AIC	-212.3695	-217.0000	-246.3839	-243.3305	-237.8542
BIC	-208.1195	-208.5000	-242.1339	-239.0805	-233.6042
CIC	-208.1195	-208.5000	-242.1339	-239.0805	-233.6042
Kendall Tau=0.4252					

Table 3: *Copula estimation results.*

Once the dependency structure is estimated, the next step is to estimate the marginal distribution functions. Considering the histograms in Figure 1, we proposed to use two EVD. Namely, we compare the log-normal distribution, that is a EVD Type I (Gumbel), with the modified Champernowne distribution<sup>1</sup>, which converges to a Pareto in the tail and therefore it is an EVD Type II (Frechet), besides the Champernowe distribution looks more like a log-normal near 0. The Champernowne distribution have been analyzed en the context of semiparametric estimation of EVD (see, for example, [3], [6], [4] and [1]). Furthermore, this distribution has

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<sup>1</sup>The cdf of the modified Champernowne distribution is:

$$F(x) = \frac{(x+c)^\delta - c^\delta}{(x+c)^\delta + (H+c)^\delta - 2c^\delta}, \quad x \geq 0,$$

with parameters  $\delta > 0$ ,  $H > 0$  and  $c \geq 0$ . The estimation of transformation parameters is performed using the maximum likelihood method described in [10].

been used to model the operational risk, where the loss distribution is similar to those analyzed in this work (see [7], [28], [8] and [22]).

In Table 4 we show the results for the maximum likelihood estimation of the marginal distributions. We can see that for Cost1 Log-normal and Champernowne have similar AIC and BIC, however for Cost2 Champernowne provides lower values of AIC and BIC.

	Log-normal	Champernowne
<b>CDFs</b>	$\int_{-\infty}^{\log x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt, \quad x \geq 0$	$\frac{(x+c)^\delta - c^\delta}{(x+c)^\delta + (H+c)^\delta - 2c^\delta}, \quad x \geq 0$
$X_1=\mathbf{Cost1}$	$\mu = 6.4437, \sigma = 1.3349,$ $AIC = 8448.8950$ and $BIC = 8452.7190$	$\delta = 1.3271, H = 677, c = 0$ $AIC = 8448.163$ and $BIC = 8453.899$
$X_2=\mathbf{Cost2}$	$\mu = 4.3755, \sigma = 1.5189,$ $AIC = 9425.1340$ and $BIC = 9428.9590$	$\delta = 1.1622, H = 88, c = 0$ $AIC = 6443.7150$ and $BIC = 6449.4510$

Table 4: Maximum likelihood estimation of marginal distributions.

For evaluating the risk of total loss we estimate the quantile at confidence level  $\alpha$  ( $q_\alpha$ ). We use Monte Carlo simulation method, the procedure is as follows:

1. We generate the pseudo-random sample  $(\hat{U}_{1i}, \hat{U}_{2i}), \forall i = 1, \dots, r$ , from the bivariate copulas whose estimate parameters are shown in Table 3.
2. Using the inverse of marginal CDFs we calculate  $(\hat{X}_{1i} = F_1^{-1}(\hat{U}_{1i}), \hat{X}_{2i} = F_2^{-1}(\hat{U}_{2i})), \forall i = 1, \dots, l$ , where the sample volume  $l$  is large.
3. We calculate  $\hat{S}_i = \hat{X}_{1i} + \hat{X}_{2i}, \forall i = 1, \dots, l$  and we estimate  $q_\alpha(S)$  empirically from the generated pseudo-sample. We generate  $l = 10,000$  samples.

In Table 5 we show the results of the estimations of  $q_\alpha$  for  $\alpha = 0.95, 0.99, 0.995, 0.999$ . On the first row of Table 5 we provide the empirical values of the  $q_\alpha(S)$  calculated with the 518

observations in the sample of the aggregate loss  $S = X_1 + X_2$  for different confidence levels  $\alpha$ , below we show the same  $q_\alpha(S)$  that have been estimated by the Monte Carlo simulation method for the five copulas considered here. We remark the importance of using an extreme value copula and extreme value marginal distributions when the data indicate this behavior.

$\alpha$	0.95	0.99	0.995	0.999
Empirical	7905.6000	24821.1400	28420.8700	92112.9300
	Log-normal			
Normal	6635.427	15628.804	20762.765	39733.894
t-Student	6547.524	16638.175	22521.175	39547.101
Gumbel	6432.017	15464.969	22011.382	40001.210
Galambos	6429.16	15471.40	22066.00	39925.67
Hustler-Reiss	6421.028	15465.126	22122.110	39841.559
	Champernowne			
Normal	7237.591	25504.175	38682.444	110082.261
t-Student	7302.165	25740.933	42223.504	117447.015
Gumbel	7264.831	23944.798	41461.743	119401.409
Galambos	7253.166	24056.946	41409.717	118982.012
Hustler-Reiss	7241.504	24103.038	41107.537	118539.744

Table 5: Quantiles of total loss.

In Table 5 we show that by using log-normal marginal distributions, the estimated quantile is below the empirical quantile for the five copulas considered here. Therefore, risk is underestimated. We also remark that the selected copula does not have much influence on the risk estimation. However, if we use Champernowne marginal distributions, which has a heavier right tail than log-normal distribution, the influence of the selected copula is not significant at

lower confidence levels (0.95 and 0.99) but it is significant for extreme confidence levels (0.995 and 0.999). As indicated by the goodness of fit measures for our data, the best selection is the Gumbel copula with Champernowne marginal distributions.

## 5 Conclusions

The test we have introduced for the adequacy of extreme value copulas lets us to determine the suitable copula, especially when the data have extreme values.

In our empirical application, the K-Plot identified a positive and increasing dependence between variables related to automobile insurance claims, and the new test we presented for extreme value copulas confirms that, in our case, we should use an extreme value copula. The results show the importance

In the selection of the marginal distribution we have considered a modified Champernowne distribution. It provides interesting results, due to its similarity to the log-normal distribution for low values of the variable and, additionally, due to its convergence to a Pareto distribution in the right tail.

When the marginal distributions have heavy right tail, as is the Champernowne distribution and if the aim is to estimate extreme quantiles, the results show the importance of testing the adequacy of the data to an extreme value copula.

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