UNIVERSITAT ROVIRA I VIRGILI

DEPARTAMENT D'ECONOMIA

CENTRE DE RECERCA EN ECONOMIA INDUSTRIAL I ECONOMIA PÚBLICA

WORKING PAPERS

Col·lecció "DOCUMENTS DE TREBALL DEL DEPARTAMENT D'ECONOMIA - CREIP"

Hours worked - Productivity puzzle: identification in fractional integration settings

Yuliya Lovcha
Àlex Pérez Laborda

Document de treball n.03 - 2013

DEPARTAMENT D'ECONOMIA – CREIP
Facultat d'Economia i Empresa

UNIVERSITAT
ROVIRA I VIRGILI
DEPARTAMENT D'ECONOMIA

CENTRE DE RECERCA EN ECONOMIA
INDUSTRIAL I ECONOMIA PÚBLICA

Edita:

Departament d'Economia
www.fcee.urv.es/departaments/economia/public_html/index.html
Universitat Rovira i Virgili
Facultat d'Economia i Empresa
Avgda. de la Universitat, 1
43204 Reus
Tel.: +34 977 759 811
Fax: +34 977 300 661
Email: sde@urv.cat

CREIP
www.urv.cat/creip
Universitat Rovira i Virgili
Departament d'Economia
Avgda. de la Universitat, 1
43204 Reus
Tel.: +34 977 558 936
Email: creip@urv.cat

Adreçar comentaris al Departament d'Economia / CREIP

Dipòsit Legal: T - 480 - 2013

ISSN edició en paper: 1576 - 3382
ISSN edició electrònica: 1988 - 0820

DEPARTAMENT D'ECONOMIA – CREIP
Facultat d'Economia i Empresa

Hours worked - Productivity puzzle: identification in fractional integration settings

Yuliya Lovcha[*] Alejandro Perez-Laborda[§]

Abstract: A recent finding of the structural VAR literature is that the response of hours worked to a technology shock depends on the assumption on the order of integration of the hours. In this work we relax this assumption, allowing for fractional integration and long memory in the process for hours and productivity. We find that the sign and magnitude of the estimated impulse responses of hours to a positive technology shock depend crucially on the assumptions applied to identify them. Responses estimated with short-run identification are positive and statistically significant in all datasets analyzed. Long-run identification results in negative often not statistically significant responses. We check validity of these assumptions with the Sims (1989) procedure, concluding that both types of assumptions are appropriate to recover the impulse responses of hours in a fractionally integrated VAR. However, the application of long-run identification results in a substantial increase of the sampling uncertainty.

JEL Classification numbers: C22, E32

Keywords: technology shock, fractional integration, hours worked, structural VAR, identification

[*] Corresponding author: Departamento de Economia, Universidad de Navarra, 31080 Pamplona, Navarra, Spain. e-mail: ylovcha@unav.es
[§] Departament d'Economia i CREIP, Universitat Rovira-i-Virgili, Spain.

For comments and suggestions we thank an anonymous referee, Maximo Camacho, Luis Alberiko Gil-Alana, Siem Jan Koopman, Antonio Moreno, Gabriel Perez-Quiros and seminar participants at various universities this paper was presented. The usual disclaimer applies.

1. Introduction

One of the implications of the Real Business Cycle (RBC) models is that the per capita hours worked rise in response to a positive technology shock. However, recent empirical works argue that this is inconsistent with the real data. Not surprisingly, these results have attracted a lot of attention since they call into a question the role attributed to a technology shock in the business cycle analysis and the ability of the RBC models to reproduce the facts presented in the data. If the empirical response of hours to a technology shock is indeed negative, the original technology-driven real business cycle hypothesis does appear to be dead and RBC models, in general, are unpromising. A prominent representative of this stream of literature is Gali (1999), who finds that the hours worked fall in response to a positive technology shock. Similar conclusions can be found in Gali and Rabanal (2004), Francis and Ramey (2005), Francis et al. (2005) and Fernald (2007), among others. From the other side, their findings have in turn been contested by Christiano et al. (2003, 2006), Canova et al (2007) and Uhlig (2004) among others.

In general, the current debate on the hours work-productivity puzzle concentrates around three issues. The first one focuses on the order of integration of the hours worked. Christiano et al. (2003), challenging the results of Gali (1999), argue that the sign and the magnitude of the responses of hours to a (positive) technology shock depend crucially on whether hours worked are assumed to be integrated of order zero (I(0)) or of order one (I(1)). Based on different testing techniques, the authors conclude that the hours worked are I(0) and hence that the hours rise in response to a technology shock. The second issue in the debate turns around the plausibility of structural vector autoregressions (SVAR) to estimate impulse responses that can comparable with theoretical responses from economic models. Chari et al. (2008), relying on the procedure described in Sims (1989), argue that the SVARs of both Gali (1999) and Christiano et al. (2003) are misspecified. The misspecification arises because the model fails the auxiliary assumption that the stochastic process for productivity and hours is well approximated by an autoregressive representation with small number of lags (a VAR(4) in both papers mentioned above). Related, the third issue revolves around the ability of the long-run (LR) identification restrictions in a SVAR to estimate reliably the dynamic responses of macroeconomic variables to structural shocks[1]. Christiano et al. (2006) show that under LR restrictions, VAR-based impulse responses exhibit some bias as a result of the difficulties in estimating the sum of the VAR parameters necessary for the LR identification. These difficulties arise from the fact that the RBC model's VAR is, in fact, infinite and econometricians fit a misspecified finite order VAR (which is the main point of the critique of Chari et al. (2008)). As a result, although individual VAR parameters and variance-covariance matrix are well estimated, there is substantial bias in the estimation of the sum of the VAR parameters necessary for the LR identification. However, the authors argue that this bias will not result in erroneous inference since the sample uncertainty is as well high when long run identification is applied. With the short-run (SR) restrictions this problem disappears and the SVARs perform remarkably well.

Regardless of the conclusions obtained, all the aforementioned studies exploit VAR approach, always assuming ex-ante the order of integration of hours worked and restricting high-lag coefficients in the VAR to be zero. Taking into account the high persistence of hours worked both in theoretical models and, especially, in the real data, these assumptions may appear to be very restrictive. The objective here is to relax the assumption on the order of integration of the hours worked in the VAR and allow for the possibility of long memory in the process that reflects the properties of the data. To do so, we rely on the fractional integration framework, which encompasses I(0) and I(1) type of processes as well as other fractionally integrated possibilities. Impulse responses in fractionally integrated VARs (VARFIMA) are identified in a similar way to standard VARs, but allowing for the additional interaction between VAR parameters and fractional integration polynomials which makes the memory of the process longer. It is important to note that the VARFIMA is an infinite VAR with slowly decaying coefficients[2]. Thus, by applying VARFIMA we do not restrict the VAR to have small finite number of lags but, rather, we restrict the coefficients of an infinite VAR to follow some definite rule.

In this work we employ five different datasets commonly used in the literature to estimate the VARFIMA model and to back up the responses of hours to a technology

[1] The first critique of the long-run identification was made by Sims (1972). More recent exponents are Faust and Leeper (1997), Uhlig (2004), Fernandez-Villaverde et al. (2007), Erceg et al. (2005), Francis et al. (2005), Christiano et al. (2006) among many others.

[2] In this framework, aggregate shocks may vanish at an hyperbolic rather than at an exponential rate.

shock under both SR and LR identification schemes. After, we analyze whether the impulse responses estimated from the VARFIMA model are comparable to the theoretical ones from RBC models. To do so, we follow Chari et al. (2008) and Christiano et al. (2006) and we make use of the Sims (1989) procedure. We choose two different RBC models that satisfy the LR and (or) the SR identification assumptions and simulate data from each of them. Thereafter, we apply the VARFIMA model to the simulated data and we uncover the impulse responses under the corresponding set of assumptions: LR identification for the data simulated from the RBC model that satisfies the long-run assumptions; and the SR identification for the data simulated from the RBC model that satisfies short-run assumptions. Then, we compare the mean of the estimated impulse responses with the theoretical responses from the corresponding RBC model.

According to our results, hours worked are fractionally integrated, possibly non-stationary mean reverting. The order of integration of productivity is always not statistically different from one. Results are robust to changes in definition of both hours and productivity, to changes in the model specification (addition of more variables into the model), to the inclusion of seasonal fractional integration and also are stable across sub-samples. The sign and the magnitude of the estimated impulse responses of hours worked to a positive technology shock depend critically on the identification assumption that was used to recover them. Thus, under the SR identification, the impulse responses are always positive and statistically significant, being very close in magnitude to the responses from the recursive RBC model. If the LR identification is applied, the responses are always negative. However, their statistical significance depends on the data choice: in all dataset the confidence intervals are very wide and the responses are not statistically significant if data is only defined for the non-agricultural sector.

The results of the Sims (1999) procedure indicate that the SR identification scheme performs very well and can be used to recover impulse responses of hours to a technology shock in a structural VARFIMA model. The LR identification is still appropriate. However, the sample uncertainty increases dramatically when it is applied. Nevertheless, the coverage rates for confidence intervals confirm that, with very high probability, VARFIMA based confidence intervals include the true value of the impulse responses no matter the identification scheme chosen by the econometrician.

Up to our knowledge, the only work on the topic using fractional integration is Gil-Alana and Moreno (2009). The authors test the order of fractional integration of hours worked in multivariate settings under different model specifications. After choosing the order of fractional integration of hours and estimating the other parameters of the model, they apply LR identification concluding that hours fall in response to a technology shock[3]. However, the authors neither check alternative identification schemes nor evaluate the validity of their identification assumptions to uncover the responses of hours from fractionally integrated models. In the light of our results, these issues are crucial to assess the response of hours to a technology shock.

The paper is organized as follows. In Section 2 we briefly review the econometrics of the VARFIMA model, describe the datasets we use, and discuss the estimation results. In Section 3 we analyze the identification issue in fractionally integrated VAR models and we evaluate the effects of technology shocks on hours worked, using structural VARFIMAs with SR and LR identification restrictions. In Section 4 we adopt the Sims procedure to evaluate the validity of the impulse responses recovered from VARFIMA model. In Section 5 we summarize the main results and provide some concluding remarks.

2. Fractionally integrated VARs

The central pillar in the Hours worked - Productivity debate is the assumption on the order of integration of hours worked. Christiano et al. (2003) argue that this assumption has a crucial effect to the sign and the magnitude of the estimated impulse responses of hours to a positive technology shock. Thus, assuming that hours worked follow an I(0) process, the contemporaneous response of hours is positive. If the hours are assumed to be I(1), as in Gali (1999), the estimated responses are negative[4].

In this work, we exploit a fractionally integrated vector autoregressive model (VARFIMA) which helps us to relax the assumption on the order of integration of hours worked. In the fractional integration framework, variables are allowed to have non-

[3] Also, the authors do not strictly apply LR identification but a sort of "*medium range*" restriction (see Section 3.2).

[4] On the basis of a series of tests, Christiano et al. (2003) conclude that hours are stationary. Chari et al. (2008) also criticize the I(1) assumption of Gali (1999) because in all economic models, hours per person are bounded and, therefore, the stochastic process for hours per person cannot literally have a unit root. However, the data on hours worked is highly persistent and the unit root hypothesis is often not rejected by unit root tests.

integer orders of integration which are not assumed but estimated with the other parameters of the model. In the following sub-sections we briefly describe the VARFIMA model and its estimation procedure. After that, we present the different datasets employed in this work and discuss the estimation results.

2.1 The VARFIMA model and its estimation

Univariate ARFIMA models can be generalized to multivariate settings leading to the VARFIMA model. More specifically, the autoregressive VARFIMA model can be written as:

$$D(L)\,y_t = v_t \tag{1}$$

$$\left(I - F_p(L)\right)v_t = w_t \tag{2}$$

where y_t is a $N \times 1$ vector of variables for $t = 1,...,T$, L is the lag operator, I is an $N \times N$ identity matrix and w_t is an $N \times 1$ vector of i.i.d errors with 0 mean and $N \times N$ variance-covariance matrix Ω. The VAR(p) process in (2) is assumed to be stationary. $D(L)$ is a diagonal $N \times N$ matrix with fractional integration polynomials on the main diagonal given by:

$$D^{(n)}(L) = (1-L)^{d_n}, \; n = 1,..., N \tag{3}$$

The scalar parameter $d_n \in [0,1]$ indicates the fractional order of integration of the series $y_{n,t}$ at frequency zero. If $d_n \in (0, 0.5)$, the series is covariance stationary but the autocovariances and responses of the variable to a shock take more time to disappear than if $d_n = 0$. If $d_n \in [0.5,1)$, the series is not covariance stationary anymore but still mean reverting, with the effect of shocks dying away in the long run. In general, the larger d_n the more persistent the variable and the stronger the policy actions required to bring it to its steady state.

The operator $D_n(L)$ can be written as the convolution of the Taylor expansions of its separate components (see Hassler (1994)):

$$D^{(n)}(L) = (1-L)^{d_n} = \sum_{k=0}^{\infty} D_{n,k} L^k, \quad D_{n,k} = \frac{\Gamma(k-d_n)}{\Gamma(k+1)\Gamma(-d_n)} \tag{4}$$

The gamma function $\Gamma(\cdot)$ satisfies $\Gamma(z+1) = z\Gamma(z)$.

The spectrum of the VARFIMA process (1) and (2) at frequency $\omega_j = \dfrac{2\pi j}{T}$, $j = 0,\ldots,T/2$, is a $N \times N$ matrix with the spectra of the variables on the main diagonal and the cross-spectra out of the main diagonal:

$$f_y(\omega_j,\theta) = (2\pi)^{-1} D\left(e^{i\omega_j}\right)^{-1} f_v(\omega_j,\theta) D\left(e^{-i\omega_j}\right)^{-1} \tag{5}$$

where i is the imaginary unit and $D\left(e^{i\omega}\right)$ and $D\left(e^{-i\omega}\right)$ are complex conjugates. The spectrum of the vector v_t is given by:

$$f_v(\omega_j,\theta) = \left(I - F_p\left(e^{i\omega_j}\right)\right)^{-1} \Omega \left(I - F_p\left(e^{-i\omega_j}\right)\right)^{-1}$$

being $F_p\left(e^{i\omega_j}\right) = F_1 e^{i\omega_j} + \ldots + F_p e^{pi\omega_j}$ and $F_p\left(e^{-i\omega_j}\right)$ is its complex conjugate.

The vector of all parameters of the model θ, contain the fractional integration parameters for all variables $\{d_n\}_{n=1,\ldots,N}$, the autoregressive parameters from the polynomial $F_p(L)$, and the parameters of the variance-covariance matrix of the vector of errors, Ω. The total number of parameters is $N + pN^2 + N(N+1)/2$.

To estimate the process given by (1) and (2) we use the approximate frequency domain maximum likelihood (Whittle estimator), proposed by Boes et al. (1989). The discussion of the multivariate version of the procedure can be found in Hosoya (1996). To derive the frequency domain likelihood function for y_t, we compute the finite Fourier transform of $y_{n,t}$, $n = 1,\ldots,N$

$$x_n(\omega, y_{n,t}) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^{T} y_{n,t} e^{-i\omega(t-1)} \tag{6}$$

An approximate log-likelihood function of θ based on y_t is given up to constant multiplication, by:

$$\ln L(\omega,\theta) = -\sum_{j=0}^{T/2} \left[\ln \det f_y(\omega_j,\theta) + tr f_y^{-1}(\omega_j,\theta) I_T(\omega_j, y)\right]$$

with the $N \times N$ periodogram matrix $I_T(\omega_j, y)$ defined as:

$$I_T\left(\omega_j, y\right) = x\left(\omega_j, y\right) x\left(\omega_j, y\right)^*$$

where $x\left(\omega_j, y\right)$ is a complex $N \times 1$ vector with entries given by (6) and $x\left(\omega_j, y\right)^*$ is its complex conjugate. For each j, the elements of the main diagonal of $I_T\left(\omega_j, y\right)$ are points of the periodogram of each of the series at frequency ω_j, which are real. The off-diagonal elements are points of the cross-periodogram, which are complex.

If the parameters of fractional integration are positive/negative at zero frequency, the spectrum tends to infinity/zero and the sample periodogram has picks/dips at this frequency. Since the likelihood function is not well defined at this point, following standard practice, we exclude it from the estimation.

2.2 Data description and preliminary analysis

To assess the response of hours we employ five different datasets which differ from each other in their measure of the hours worked and productivity.

Dataset FR. Francis and Ramey (2005) use a new measure of hours per capita. The authors argue that the hours per capita measured in a standard way (dividing private hours by the non-institutional population aged 16 and over), are significantly affected by low frequency demographic and institutional trends and, as a result, display significant low frequency movements. They develop a more sophisticated measure of population available for work in the private sector and use it to calculate hours per capita in that sector. The FR dataset we use is the updated version of the data employed in their paper and it can be found in the official web-page of Ramey, V.A. Data is quarterly and covers from 1947:1 to 2007:4. Hours worked are defined as the natural logarithm of the ratio hours worked in the private sector to the new measure of population available to work in the private sector. Productivity is the natural logarithm of the output per hour in the private sector. This dataset is especially interesting since the authors find that a positive technology shock (identified through LR restrictions) leads always to a decrease in hours, no matter one assumes that hours per capita are stationary or not.

Dataset GR. Gali and Rabanal (2004) use U.S. quarterly data for the period 1948:1 to 2002:4. The source of the data is the Haver USECON database. The series of output in this database corresponds to non-farm business sector output (LXNFO). Labor input series is hours of all persons in the non-farm business sector (LXNFH). Output and hours series are expressed in per capita terms, using the measure of civilian non-institutional population aged 16 and over (LNN). All the series are converted into the logarithm form.

Dataset CEV. This is the data employed by Christiano et al. (2003). It is drawn from DRI Basic Economics database and covers period from 1948:1 to 2001:4. They use business labor productivity (LBOUT) as a measure of productivity and business hours (LBMN) divided by civilian population over the age of 16 (P16) as a measure of hours. All the series are converted into logarithm form.

The following two datasets are constructed by us using data collected from the Federal Reserve Bank of St. Louis database (FRED)[5]. These two datasets run from 1948:1 to 2009:4 covering a longer period than the other popular datasets in the literature.

Dataset A is similar to the data of Christiano et al. (2003), and contains data from all sectors, including farm sector. Total business productivity is the log of the output per hour of all persons (OPHPBS) and hours worked is the log of the ratio of the business hours of all persons (HOABS) to the civilian non-institutional population over the age 16 (CNP16OV).

Dataset B is close to the dataset of Gali (1999). The only difference is the definition of hours worked. Gali (1999) uses the log of total employee-hours in non-agricultural establishments, while we perform analysis using hours per capita. We use per capita specification because the hours worked in the business cycle models are usually in per capita terms. Non-agricultural business sector productivity is the log of OPHNFB series in FRED, hours worked are constructed by subtraction the log of the civilian non-institutional population over the age 16 (CNP16OV) from the log of the non-farm business sector hours of all persons (HOANBS).

In the datasets A and B "the civilian non-institutional population over the age 16" is converted to quarterly series by taking the simple average of the monthly observations inside the quarter. Except of population, all the series in all datasets are

[5] http://research.stlouisfed.org/fred2/

seasonally adjusted. The population series is not seasonally adjusted since seasonal adjustment is not applicable for this series.

Figure 1 plots hours worked and productivity series for all data sets. Data is quarterly, transformed to natural logarithms, in levels. We do not plot series from the CEV dataset because they are very similar to ones from the dataset A, but cover shorter period[6].

We perform a set of commonly used unit root tests for the hours worked series in each dataset[7]. The results of the testing procedure may be found in the Table 1. The null hypothesis states always that the underlying process is a unit root (with or without drift). As it can be seen in the table, the results are not conclusive. In some of the cases the null hypothesis cannot be rejected at usual levels of significance, but the p-values of the tests are low. However, the null hypothesis is rejected by the tests corrected for serial correlation in residuals. On the basis of the results described above, there is no certainty about the order of integration of the hours worked. Likely, the order of integration is not integer and lies between zero and one. Thus, the estimation of the order of (fractional) integration of the hours worked is interesting by itself.

2.3 Estimation results

Prior to estimation, we take first difference of the data. This is required to receive estimates of the parameter of fractional integration inside the stationary region $d_i \in (-0.5, 0.5)$ and is indicative of the strong persistence of both hours and productivity series in all datasets. It is very important to note that taking first difference is not equivalent to the assumption that the variables are I(1). In the VARFIMA framework, the orders of integration of the variables are estimated, not assumed. If a variable is over-differenced, its order of fractional integration at zero frequency is expected to be negative. The results of estimation are summarized in the Table 2. The estimated orders of integration of the series in levels (as they appear in the table) are obtained by adding one to the estimated value in differences: $1+\hat{d}_i$. Standard errors of the coefficients are computed by numerical evaluation of the Hessian matrix and are presented in

[6] The difference between these series is due to not coinciding last revision period.
[7] The unit root hypothesis in productivity is not rejected by any test. These results are not discussed here since there is agreement in the literature on the existence of a unit root in that series.

parenthesis. The orders of autoregression for all models are chosen by Schwarz information criteria.

The columns labeled with "1" contain the estimated parameters of the VARFIMA model. As expected, the hours series exhibit long memory: the estimated coefficient of fractional integration lies between 0.6-0.7 in all datasets. Furthermore, these estimates are statistically different from both 0.5 and 1. Thus, hours are found to be non-stationary but still mean reverting. In line with previous studies, the coefficient of fractional integration of the productivity series is not statistically different from one in any dataset considered. Consequently, we estimate a restricted version of the VARFIMA model assuming that productivity is I(1)[8]. The estimation results of the restricted version are presented in the Table 2 in columns labeled with "2".

3. The response of hours to a technology shock

In this section we first analyze identification in fractionally integrated autoregressive processes. After, we compute the impulse responses of hours to a technology shock from the estimated VARFIMA model in all datasets.

3.1 Structural and reduced form models

We want to analyze identification in the following structural model:

$$AD(L)y_t = u_t \tag{7}$$

$$\left(I - G_p(L)\right)u_t = \varepsilon_t \tag{8}$$

where A is the matrix of structural parameters. Let $D(L)$ to be the diagonal matrix with long memory polynomials defined as in (3). The $N \times N$ matrix $G_p(L)$ contains the short memory autoregressive polynomials of order p and ε_t is a $N \times 1$ vector of structural shocks with 0 mean and variance-covariance matrix V .

Substituting (7) into (8), premultiplying both sides of the resulting expression by A^{-1} and simplifying we get the structural MA(∞) representation of y_t :

$$y_t = D(L)^{-1}\left[I - A^{-1}G(L)A\right]^{-1}A^{-1}\varepsilon_t \tag{9}$$

[8] More interestingly, this assumption simplifies the long run identification in VARFIMA models significantly as will be clear in the next section.

The reduced-form MA(∞) representation of the model given by (1) and (2) is obtained by substitution of (1) into (2). After arrangement, the resulting expression is:

$$y_t = D(L)^{-1}\left[I - F_p(L)\right]^{-1} w_t \tag{10}$$

Substitution of $w_t = A^{-1}\varepsilon_t$ (follows from (9) and (10)) into the reduced-form MA(∞) representation of y_t given by (10) results in:

$$y_t = D(L)^{-1}\left[I - F_p(L)\right]^{-1} A^{-1}\varepsilon_t = \Lambda(L)\varepsilon_t \tag{11}$$

The impulse responses of variables to structural shocks in this model are given by the coefficients of $\Lambda(L)$ in (11). To find these coefficients we make the convolution of the Taylor expansions of the separate components of $\Lambda(L)$.

3.2 Identification in fractionally integrated VARs

The identification in fractionally integrated models is achieved in a similar way to the standard VAR. However, it has some important nuances that must be discussed.

Without loss of generality we assume that the model in (1) and (2) is bi-variate[9]. We define the first variable in the model to be the first difference of the logarithms of the hours worked and the first shock the shock to hours (or demand shock). The second variable is the difference of logarithms of productivity and its corresponding shock the technology shock.

The quarterly structural model (9) has $N + N^2 + N^2 p + N(N+1)/2$ parameters: one coefficient of fractional integration for each of the N variables, N^2 structural parameters from matrix A, $N^2 p$ autoregressive parameters from matrix $G(L)$ and $N(N+1)/2$ parameters from the variance-covariance matrix V. The reduced-form model (10) has only $N + N^2 p + N(N+1)/2$ parameters. As a result, to reach identification in the bi-variate structural model (N = 2) we have to make $N^2 = 4$ additional identification restrictions.

A standard practice is to assume that the structural shocks are orthogonal and have been scaled by their standard deviations which can be expressed by the assumption

[9] The generalization of the model for N variables is straightforward.

$V = I$. In such a way, we are making $N(N+1)/2=3$ restrictions, but still we require one additional assumption.

Let the matrix of contemporaneous impulse responses A^{-1} to be:

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The impulse responses of variables to structural shocks in this model are given by:

$$\begin{aligned}
\Lambda_{11}(L) &= \Psi^{(1)}(L)\left(Q_{11}(L)a + Q_{12}(L)c\right) \\
\Lambda_{12}(L) &= \Psi^{(1)}(L)\left(Q_{11}(L)b + Q_{12}(L)d\right) \\
\Lambda_{21}(L) &= \Psi^{(2)}(L)\left(Q_{21}(L)a + Q_{22}(L)c\right) \\
\Lambda_{22}(L) &= \Psi^{(2)}(L)\left(Q_{21}(L)a + Q_{22}(L)c\right)
\end{aligned} \tag{12}$$

where $\Lambda_{ij}(L) = \Lambda_{ij0} + \Lambda_{ij1}L + ... + \Lambda_{ijk}L^k + ...\,;\ i, j = 1,2;\ k \geq 0$ are infinite polynomials and Λ_{ijk} denotes the impulse response (IRF) of the variable *i* to the shock *j* at lag *k*. $Q_{ij}(L)$ are infinite order lag polynomials that can be found by solving $\left(I - F(L)\right)^{-1} = Q(L)$ and $\Psi^{(i)}(L)$ is the inverse of the fractional integration polynomial $D^{(i)}(L)$.

Recall that the variables in (1) are defined in first differences, as it was required to insure that the estimated coefficients of fractional integration d_i lie within the stationary interval. To find the IRFs of the variables in levels, one has to sum the IRFs of the differenced variable up to the lag of interest: $\bar{\Lambda}_{ijk} = \sum_{l=0}^{k} \Lambda_{ijl}$, where $k \in [1, \infty)$. The responses of variables in levels are the coefficients of the polynomial $\bar{\Lambda}_{ij}(L) = \bar{\Lambda}_{ij0} + \bar{\Lambda}_{ij1}L + ... + \bar{\Lambda}_{ijk}L^k + ...$. Contemporaneous IRF $\bar{\Lambda}_{ij0} = \Lambda_{ij0}$ $(L=0)$ are given by the entries of the matrix A^{-1}. The long-run response of a variable i to a shock j is given by $\bar{\Lambda}_{ijk} = \sum_{l=0}^{k} \Lambda_{ijl}$ with $k \to \infty$ and is commonly notated as $\Lambda_{ij}(1)$.

The equations relating the reduced and structural shocks $w_t = A^{-1}\varepsilon_t$ are given by:

$$\begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} = \begin{bmatrix} a\varepsilon_{1,t} + b\varepsilon_{2,t} \\ c\varepsilon_{1,t} + d\varepsilon_{2,t} \end{bmatrix}$$

Since w_t is estimated by the reduced VARFIMA and $\operatorname{var}(\varepsilon_{1,t}) = \operatorname{var}(\varepsilon_{2,t}) = 1$ by the assumption $V = I$, the contemporaneous IRFs of the variables to shocks (the terms a, b, c and d in the matrix A^{-1}) can be found by solving the following system of equations:

$$\begin{aligned} \sigma_1^2 &= a^2 + b^2 \\ \sigma_2^2 &= c^2 + d^2 \\ \sigma_{12} &= ac + bd \end{aligned} \tag{13}$$

This system of three equations has four unknowns. To identify this system we require one additional restriction.

3.2.1 Short-run (SR) identification

The easiest way to make the last identification assumption is to assume that the hours are not influenced by the technology shock contemporaneously. This type of identification is usually called in the literature as the short-run (SR) or Sims (1972) identification. To apply this assumption one has to restrict the matrix of the contemporaneous responses A^{-1} to be lower-triangular $(b = 0)$.

After solving the system (13) and assuming $b = 0$, the contemporaneous responses are defined up to a sign by:

$$[a,b,c,d] = \left[\pm\left(\sigma_1^2\right)^{1/2}, 0, \frac{\sigma_{12}}{a}, \pm\left(\sigma_2^2 - c^2\right)^{1/2} \right] \tag{14.}$$

To identify the responses of hours to a technology shock we assume that the contemporaneous response of productivity to technology shock is positive: $d > 0$ [10]. In this way the responses of hours to a positive technology shock are given by $\Lambda_{12}(L) = \Psi^{(1)}(L) Q_{12}(L) d$. Since fractional integration does not influence directly the identification process, the SR assumption can be applied in the same way to the standard VAR.

[10] Under the short-run identification, the Cholesky decomposition of the variance-covariance matrix of the reduced-form errors can be applied. It corresponds to the assumption that the contemporaneous impulse response of productivity to a positive technology shock is positive.

3.2.2 Long-run (LR) identification

Another common way to make the last restriction is the Blanchard-Quah (1989) procedure or long-run identification (LR). The restriction is that in the long-run, the labor productivity is driven by the technology shock only. It means that the long-run IRFs of the productivity to a non-technology shock (demand shock) are equal to zero:

$$\Lambda_{21}(1) = \Psi^{(2)}(1)\left(Q_{21}(1)a + Q_{22}(1)c\right) = 0 \tag{15}$$

If productivity is fractionally integrated, the application of the LR identification restriction is not trivial[11]. Notice that the LR identification restriction (15) does not depend on the order of integration of hours worked, just of productivity. If the coefficient of fractional integration of productivity is smaller than one: $1+\hat{d}_2 < 1$ (the estimated coefficient in differences $\hat{d}_2$ is negative), the term $\Psi^{(2)}(1)$ is equal to zero and the coefficients *a*, *c* cannot be identified because $\Lambda_{21}(1)$ is zero for all *a*, *c*. Also, in this case, the response of productivity to its own shock will converge to zero in the long run, contradicting the implications of the RBC model, where the long run response of productivity to technology shock is strictly positive. From the other side, if the coefficient $\hat{d}_2$ is positive (productivity has an order of integration higher than one), then $\Psi^{(2)}(1) = \infty$ and the LR scheme cannot be applied without truncation of the infinite sum of the terms of the polynomial $\Lambda_{21}(L)$ up to some lag k << ∞, as in Gil-Alana and Moreno (2009). However, this no longer can be understood as the application of the LR assumption but a sort of "medium range" restriction.

Since the order of integration of productivity is not statistically different from one in any dataset, we can overcome these problems by computing the responses of hours to a technology shocks from the restricted fractional models only. It is, we restrict productivity to be I(1) in the VARFIMA but we leave hours worked unrestricted. In this case $\Psi^{(2)}(1) = 1$ and the long run restriction (15) becomes:

$$\Lambda_{21}(1) = Q_{21}(1)a + Q_{22}(1)c = 0 \tag{16}$$

[11] See Tschernig et al. (2010) for details about LR identification in fractionally integrated systems.

From the last expression, we can find the contemporaneous response of productivity to a positive demand shock as:

$$c = af \tag{17}$$

where $f = -Q_{21}(1)/Q_{22}(1)$. Now we have a system of four equations with four unknowns given by (13) and (17)[12]. The solution of this system is given by:

$$[a,b,c,d] = \left[\pm\left(\frac{\sigma_1^2\sigma_2^2 - (\sigma_{12})^2}{-2\sigma_{12} f + \sigma_1^2 f^2 + \sigma_2^2} \right)^{1/2}, \frac{(\sigma_{12} - a^2 f)}{d}, af, \pm\left(\sigma_2^2 - a^2 f^2\right)^{1/2} \right] \tag{18}$$

The contemporaneous IRFs are identified up to a sign. The sign assumption we make is that the long-run response of productivity to a technology shock is positive: $\Lambda_{22}(1) > 0$. The sign of the parameter a does not influence the response of hours to a productivity shock.

The LR identification assumption is compatible with a wide range of economic models and focuses on their long-run properties. However, LR restrictions have very serious limitations and have been strongly criticized since the seminal work of Sims (1972). The main of these limitations is that they require the estimation of the sum of the responses of the variables to shocks. This is computed as the inverse of the matrix of the sum of autoregressive coefficients. Thus, even a small bias in the sum of autoregressive coefficients is substantially magnified by the application of a non-linear transformation. Also, the sum of autoregressive coefficients by itself cannot be precisely estimated by two reasons. First, model's VAR have an infinite number of lags which is not feasible for the econometrician. Second, it is not possible to approximate accurately the long-run behavior of an economic time series from the short span of data usually available. According to Faust and Leeper (1997) such structural VARs are not expected to be realistic in finite samples. The same critique applies to whatever model estimated with real data, including VARFIMA. Contrary, SR restrictions are applied directly to the estimated variance-covariance matrix of the reduced form disturbances and are

[12] By a matter of robustness, we have also employed truncation without restricting productivity to be I(1) obtaining similar results.

immune to the majority of problems that suffers LR identification. However, they tend to be model specific[13].

3.3 Estimated IRFs

The responses of the hours worked to the technology shock from the restricted VARFIMA model (where productivity is assumed to be I(1)) together with the 95% confidence intervals are plotted in the Figure 2. The confidence intervals for the IRFs are computed by multivariate non-parametric bootstrap in frequency domain (see Berkowitz and Diebold (1998)[14]). To compare our results with other's in the literature, we also plot the IRFs from two VAR(4) models with different assumption on the order of integration of hours worked (I(0) and I(1)). We compute IRFs for all models under both sets of identification assumptions: SR and LR.

Under SR identification, the *contemporaneous* response of hours to a technology shock is equal to zero in all models by assumption (b=0). The estimated VARFIMA response of hours to a positive technology shock after one period is positive and statistically significant in all datasets. The speed of decline of the impulse responses depends on the estimated coefficient of fractional integration of hours worked: the higher $\hat{d}_1$ the slower the decline of the impulse responses. The IRFs uncovered from both VAR models are as well positive and significant in all datasets analyzed. The responses from VAR(4) with I(1) hours do not approach zero in the long run because hours are non-stationary in this model. It worth also to notice that the responses recovered with VARFIMA are much closer in magnitude to the theoretical ones arising from RBCs, as it will be clearer in Section 4.

Under LR identification, results are less homogeneous. The estimated VARFIMA responses of hours are negative. However, they are only statistically significant in the datasets FR, CEV and A. In fact there is a huge increase of the sampling uncertainty associated to the LR scheme and the confidence intervals are much wider than those obtained with SR identification. In line with the results of Christiano et al. (2003), the estimated responses of hours to technology shock of the two

---

[13] We thank the referee for pointing out the existence of this "trade-off".

[14] To compute confidence intervals we produce 500 bootstrap replications treating the estimated model as the true data generating process. Conditions on the spectral density of VARFIMA process for the application of the bootstrap are satisfied for all frequencies except for frequency zero. Since we exclude frequency zero from the estimation, we do not bootstrap the periodogram for this frequency.

VAR models are of opposite sign and very different from the responses of VARFIMA. If hours are assumed I(0), the estimated VAR responses uncovered with LR scheme are positive and strong, as in Christiano et al. (2006) or Chari et al. (2008). Opposite, when hours are assumed I(1), the estimated responses become negative in all datasets.

Summarizing, when relaxing the assumption on the order of integration of hours worked, the estimated responses obtained with the SR identification are positive and statistically significant in all datasets. Further, the responses are very similar to the ones derived from RBC models. LR identification results in negative but often not statistically significant responses. In fact, the responses are never significant if productivity and hours are defined only for the non-agricultural sector. We analyze the robustness of these results in the following subsection.

3.4 Robustness checking

We perform a robustness checking of the previous results along three different dimensions. First we analyze the stability of the results across sub-samples. Second we extend the estimation framework to account for seasonal fractional integration. Finally, we investigate whether the inclusion of more variables into the model has effect on the estimated IRFs.

3.4.1 Stability across sub-samples

Chari et al. (2008) argue that that the responses of hours to a technology shock are not stable across sub-samples. As a robustness checking exercise, we produce the estimation in the sub-sample starting from 1959:1 for all datasets considered. The estimated VARFIMA parameters of fractional integration at zero frequency of hours are 0.58, 0.64, 0.60, 0.57, and 0.63 in the datasets FR, GR, CEV, A, and B, correspondingly. The estimated coefficients in the sub-sample are slightly lower than in the full data sample, but still they point out the non-stationary mean reverting behavior of hours worked. The estimated coefficients of productivity in all datasets are again not statistically different from one. As a consequence, we conclude that the results on the orders of fractional integration of the variables are stable across sub-samples. The responses of the hours worked to the technology shock from the restricted VARFIMA are very similar to the ones obtained with the full sample and are not reproduced in the article to conserve space[15]. However, it is important to note that the responses recovered with LR assumption in the dataset FR become statistically not significant. In this way, if the SR assumption is applied, the IRFs in the sub-sample starting from 1959:1 are positive and statistically significant in all datasets analyzed. The responses uncovered with LR identification are negative but with very wide confidence intervals, and statistically not significant in three out of five datasets (FR, GR, and B).

3.4.2 Seasonal fractional integration

The VARFIMA model can be expanded to a more general case that includes fractional integration not at the zero frequency only, but also at seasonal frequencies (VARFISMA)[16]. Seasonal fractional integration is a very common feature of many seasonally not adjusted economic time series documented in Porter-Hudak (1990), Gil-Alana and Robinson, (2001), etc. However, even if the data is seasonally adjusted, it still may be seasonally fractionally integrated with negative orders of integration due to seasonal over-differencing[17]. The omission of the negative seasonal fractional integration may result in biased estimates of the coefficient of fractional integration at zero frequency (Lovcha and Perez-Laborda (2010)). Also, the seasonal fractionally integrated model will help us to account for the influence of high frequency (with period less or equal to one year) movements that may be present in the data, which, as noted in Francis and Ramey (2005), can influence the responses of hours to a technology shock. The estimation results of the VARFISMA model for the five datasets are also provided in Table 2 in the columns labeled with "3". In line with the results from the previous section, the estimated order of integration of the productivity series is again not statistically different from one in any dataset considered. The coefficients of fractional integration at seasonal frequencies are negative and significant for the hours worked series but not for productivity in any dataset. Consequently, we estimate a restricted version of the VARFISMA model assuming that productivity is I(1) at zero frequency

---

[15] These results may be obtained upon request to the authors.

[16] For quarterly data, the flexible seasonal autoregressive VARFISMA specification can be defined as in (1) and (2), with the fractional integration polynomials on the diagonal of the N×N matrix D(L) now given by $D_n(L)=(1-L)^{d_{n0}}(1+L)^{d_{n2}}(1+L^2)^{d_{n1}}$, where d_{n0}, d_{n2}, d_{n1} are the parameters of fractional integration for the series y_{nt} at frequency zero and at seasonal frequencies $\pi/2$ and π correspondingly.

[17] Seasonal adjustment leads often to seasonal over-differencing, and produces the so-called dips in the sample periodogram at seasonal frequencies that correspond to negative seasonal fractional integration (see e.g. Lovcha et al (2012)).

and I(0) at seasonal frequencies. The results of the estimations of the restricted version of the VARFISMA model are presented in the Table 2 in the columns labeled with "4".

As can be seen in the table, although the hours worked exhibit long memory in both specifications, the main difference in the results between the two fractionally integrated models (VARFISMA vs. VARFIMA) is that the estimated parameters of fractional integration at zero frequency of hours worked are always smaller for the VARFISMA specification. As can be seen in Figure 2, this is important to analyze the speed of decline of the impulse responses of hours, since the impulse responses recovered from VARFISMA will decline considerably faster than the ones from VARFIMA. Nevertheless, the IRFs from VARFISMA are always inside the VARFIMA confidence intervals. Overall, the omission of seasonal fractional integration does not seem to alter previous results significantly.

3.4.3 Model specification

To check the robustness of the results to changes in the model specification, we construct two expanded datasets, **Dataset A3** and **Dataset B3**, including the investment-output ratio to datasets A and B respectively[18]. The choice of the third variable is dictated by two considerations. First, according to Chari et al. (2008) the inclusion of a capital like variable may improve the results of the empirical model. Second, as the same authors show, the investment-output ratio is the only capital-like variable that produces invertible moving average representation of the RBC model in state space form. That is why the IRFs from the empirical model with the investment-output ratio can be compared with the ones from RBC. We estimate IRFs from a VARFIMA model with three variables for the datasets A3 and B3.

The results on the order of fractional integration of hours are robust to the specification change with the coefficients of the fractional integration lying in the same range: 0.6-0.7. The orders of integration of productivity and the investment-output ratio appear to be not statistically different from one. As a result, we restrict both variables to be I(1). To identify the IRFs with SR assumptions in a three variables system we assume that neither the hours nor the investment-output ratio are influenced by the

[18] The investment-output ratio is computed as the natural logarithm of the ratio of real gross private domestic investment (GPDIC96 in FRED) to the corresponding series of output. The later is computed as the product of productivity times hours worked from the corresponding two-variable dataset.

technology shock contemporaneously; and the hours are not influenced by the shock to investment-output ratio[19]. The long-run identification is produced assuming that the technology shock is the only shock that influences productivity in the long run; and the investment-output ratio is not influenced by the shock to hours in the long-run[20].

Figure 3 plots the IRFs uncovered from the three-variable VARFIMA together with the responses and confidence intervals from the corresponding two-variable model. As can be seen in the figure, the responses from the three-variable model are always inside the confidence intervals of the VARFIMA with two variables and very close to each other. We also plot in the same figure the responses computed from the two standard VARs (with the same three variables), with the I(0) and I(1) assumptions on hours worked. As can be seen in the Figure 3, the results are not influenced significantly by the inclusion of the third variable, being analogous two the ones plotted in the Figure 2.

Overall, an interesting conclusion arises from the empirical results of Section 2: the sign and the magnitude of the responses of hours to a technology shock depend crucially on the identification assumption employed to uncover them. Given the robustness of the results to the definition of hours and productivity, to the inclusion of seasonal fractional integration and to the changes in model specification, the only possible reason to get a negative response of hours seems to be the application of the LR assumption. Moreover, even if it is the case, the negative responses are statistically different from zero only in three datasets from five in the full sample and in two datasets from five in the sub-sample starting from 1959.

4. Simulation study

Given that the sign and magnitude of the responses of hours to a technology shock in all datasets analyzed depend crucially on the assumptions employed to identify them, we check the validity of the identification assumptions in a structural fractionally integrated autoregressive settings. To do so, we employ the procedure described in Sims

[19] In other words, to apply the Cholesky decomposition to the variance-covariance matrix of the (fractional) VAR disturbances, we order hours as the first variable in the vector, followed by the investment-output ratio and, finally, productivity. However, the results do not change if we change the order of hours and investment-output ratio in the vector.

[20] Results on IRFs do not change if we assume instead that hours are not influenced by investment-output ratio shock in the long run.

(1989) and after applied by Christiano et al. (2006) and Chari et al. (2008) among others.

4.1 The Sims procedure

The Sims procedure (Sims (1989)) can be divided into three steps. First, artificial datasets are simulated from a parameterized RBC model. Second, the estimation procedure examined is applied to this artificial datasets and the impulse responses are recovered with the identification assumptions satisfied in the model. Third, the mean of the impulse responses estimated from the artificial datasets is compared with the theoretical responses from the RBC model. If they are close enough to each other, the estimation procedure and the set of the identification assumptions applied are appropriate to uncover the impulse responses in the structural empirical model. The basic idea of the procedure is the following. The researcher knows that the DGP is a particular RBC model. Under some conditions (satisfied in the model we use), this RBC has infinite order VAR representation[21]. Since an infinite VAR is not feasible model for estimation, the researcher has to apply a set of auxiliary assumptions. For example, to assume that this infinite VAR is well represented by a VAR(4) with I(1) hours worked as in Gali (1999), by a VAR(4) with I(0) hours worked as in Christiano et al. (2003), or by a VARFIMA with fractionally integrated hours, as in this paper. All three are different versions of a restricted infinite VAR. The first two models restrict the order of integration of variables and the number of lags of the process. The last one relaxes these two assumptions, but it still restricts the coefficients of the infinite VAR to be of a particular form[22]. The objective here is to check if the applied auxiliary and identification assumptions are reasonable to uncover the impulse responses of the variables to structural shocks. The weakness of the approach is that it is difficult to separate the effect of the auxiliary and the identification assumptions to the quality of the estimated responses. To overcome this problem, we follow Christiano et al. (2006) and use two versions of the RBC model which differ in the timing assumptions. The first version is a standard RBC where all time t decisions are taken after the realization of the shocks. This model satisfies LR identification assumptions. The second version is a recursive RBC, where the labor decision at period t is made before the realization of

[21] For the details, see Fernandez-Villaverde et al. (2007) and Christiano et al. (2006).

[22] VARFISMA model is a restricted infinite VAR with coefficients that can be found multiplying the matrix of infinite long memory polynomial $D(L)$ by the matrix of finite low lag short memory polynomials (I-$F(L)$).

the period's t technology shock. The recursive version satisfies both LR and SR identification. Making use of the recursive version of the RBC in the Sims procedure, we can separate the effect of the auxiliary assumptions from the effect of the identification assumption. Given that to the VARFIMA model estimated using data from the recursive RBC can be applied both identification schemes, results will be both data and model invariant. We briefly describe the two versions of the model in the following subsection.

4.2 RBC model

According to our previous analysis, the inclusion of the third variable does not alter the results. We henceforth focus on the two-variable case and employ the two shocks version of the RBC model of Christiano et al. (2006) as the true DGP[23].

The representative agent maximizes expected utility over per capita consumption, c_t, and per capita hours worked, l_t:

$$E_0 \sum_{t=0}^{\infty} \left(\beta(1+\gamma)\right)^t \left[\ln c_t + \varphi \frac{(1-l_t)^{1-\sigma} - 1}{1-\sigma} \right]$$

subject to the household budget constraint:

$$c_t + (1-\tau_x) i_t \leq (1-\tau_{l,t}) w_t l_t + r_t k_t + T_t$$

where capital accumulation is given by:

$$i_t = (1+\gamma) k_{t+1} - (1-\delta) k_t$$

and the resource constraint:

$$c_t + i_t \leq y_t$$

[23] Also, focus on the two-variable model help us to alleviate the huge computational burden associated to the estimation of the VARFIMA model which is crucial given the high number of simulations required for the exercise. Additionally, results using data simulated from a three-variable RBC are not expected to be different to the ones provided in the text. Chari et al. (2008) show that the inclusion into the VAR of the log investment-output ratio generated by RBC with three shocks does not change the results since the highest eigenvalue of the decay matrix in the infinite VAR representation is as higher as the corresponding one from the two-variable case. Nevertheless, by a matter of robustness, we evaluate the responses simulating data from a three-variable version of the RBC model of Christiano et al. (2006) and producing estimation with the two-variable VARFIMA. This is done to evaluate whether the inclusion additional state variables produces finer results, even when restraining the VAR to incorporate only two variables, as it is sometimes the case. The results are analogous to the ones obtained using data simulated from the two-variable RBC model and not provided in the text to save space, but they are available from the authors upon request. We thank the referee for suggesting this robustness checking.

Here, k_t is the per capita capital stock at the beginning of period t, w_t and r_t are the wage and the rental rate on capital at t, correspondingly, τ_x is an investment tax, $\tau_{l,t}$ is the tax rate on labor income, $\delta \in (0,1)$ is the depreciation of capital, γ is the growth rate of the population, T_t is lump-sum taxes, σ is the curvature parameter in the utility function, β is the discount factor and, φ is the preference parameter.

The representative firm's production function is:

$$y_t = k_t^{\alpha}\left(Z_t l_t\right)^{1-\alpha}$$

where Z_t is the time t state of the technology and $\alpha \in (0,1)$.

Non-stationary technology process represented by a random walk:

$$\ln Z_t = \mu_z + \ln Z_{t-1} + \sigma_z \varepsilon_t^z$$

that can be rewritten as:

$$\ln \frac{Z_t}{Z_{t-1}} = \ln z_t = \mu_z + \sigma_z \varepsilon_t^z$$

The process for the demand shock (labor tax shock in this paper) is represented by stationary (but persistent) AR(1) process.

$$\tau_{l,t} - \tau_l = \rho_l\left(\tau_{l,t-1} - \tau_l\right) + \sigma_l \varepsilon_t^l$$

The shocks ε_t^z and ε_t^l are i.i.d random variables orthogonal to each other with zero mean and variance equal to one. The constants μ_z and τ_l are the mean growth rate of technology and the mean labor tax rate, correspondingly. The autoregressive coefficient ρ_l is restricted to be less than one in absolute value. Under these assumptions on the dynamic of the technology and the labor tax rate, the hours per capita should be stationary.

Following Christiano et al. (2006), we consider two versions of the RBC model with different timing assumptions: the standard (or non-recursive) and the recursive version. In the standard version, all time t decisions are made after the realization of the time t shocks. The log-linearized equilibrium laws of motion for capital and hours in this version of the model can be written as:

$$\begin{aligned} \ln l_t &= \xi_0 + \xi_k \ln \tilde{k}_t + \xi_z u_t + \xi_l \overline{\tau}_{l,t} \\ \ln \tilde{k}_{t+1} &= \gamma_0 + \gamma_k \ln \tilde{k}_t + \gamma_z u_t + \gamma_l \overline{\tau}_{l,t} \end{aligned} \tag{19}$$

where $u_t = \ln z_t - \mu_z$, $\overline{\tau}_{l,t} = \tau_{l,t} - \tau_l$ and $\tilde{k}_{t+1} = k_{t+1}/Z_t$ [24].

From the two equations above and the log-linearized production function, it is clear that the technology and labor tax shocks only have a temporary impact on $\ln l_t$ and $\ln k_t$, but the technology shock ε_t^z has a permanent effect on log labor productivity. Thus, the technology shock is the only shock that influences productivity in the long run, and the standard version of the model satisfies the LR assumption.

In the recursive version, the labor decision in period t is made before the realization of the period's t technology shock. The log-linearized equilibrium laws of motion for capital and hours can now be written as follows:

$$\begin{aligned} \ln l_t &= \xi_0 + \xi_k \ln \tilde{k}_t + \xi_z u_{t-1} + \xi_l \overline{\tau}_{l,t} \\ \ln \tilde{k}_{t+1} &= \gamma_0 + \gamma_k \ln \tilde{k}_t + \gamma_z u_t + \gamma_l \overline{\tau}_{l,t} + \gamma_{z,1} u_{t-1} \end{aligned} \tag{20}$$

As before, the only shock that affects the labor productivity in the long run is the technology shock, and again LR identification is satisfied. However, due to the timing assumption, the labor decision at t is not affected by the time t technology shock, and SR identification can also be used to uncover the response of hours.

The RBC impulse responses of hours to a technology shock are calculated as follows. The technology shock is set $\varepsilon_0^z = \Delta > 0$ at period 0 (steady state), and zero afterwards ($\varepsilon_t^z = 0$ for $t \geq 1$). The labor tax shock is switched off ($\varepsilon_t^l = 0, \forall t$). The responses of hours to the technology shock can be calculated recursively from the laws

[24] Details on the variables' transformation, log-linearization, model' solution, and state space form representation may be found in the Appendix A on the authors' web-pages.

of motion of capital and hours given by equation (19) for the standard version of the model and equation (20) for the recursive[25].

4.3 Parameterization and simulation setup

As in Christiano et al. (2006), we employ standard parameterization of the RBC model: $\beta = 0.98^{1/4}$, $\alpha = 0.33$, $\delta = 1-(1-0.06)^{1/4}$, $\varphi = 2.5$, $\gamma = 1.01^{1/4}-1$, $\tau_x = 0.3$, $\tau_l = 0.242$, $\mu_z = 1.016^{1/4}-1$, $\rho_z = 0$, $\sigma = 1$.

The laws of motion for the exogenous shocks are given by:

$$\begin{aligned} \ln z_t &= \mu_z + 0.00953\varepsilon_t^z \\ \tau_{l,t+1} &= (1-0.986)\tau_l + 0.986\tau_{l,t} + 0.0056\varepsilon_{t+1}^l \end{aligned}$$

We simulate 500 artificial datasets from the standard and recursive versions of the parameterized RBC. Initial state (or period t=0) is the steady state, the number of observations in each simulated series is 240, corresponding to 60 years[26]. The errors ε_t^z and ε_t^l are generated from the standard normal distribution.

For each of the simulated datasets we produce estimation with the VARFIMA. As in the real data analysis, we restrict the productivity series generated by the RBC models to be I(1) and we assume that the process is well approximated by the VARFIMA model with one lag in autoregressive, as chosen by Schwarz criteria in the empirical part. We recover the impulse responses under the same set of assumptions that are satisfied in the DGP. Hence, we apply both identification schemes when we produce estimation with the data simulated from the recursive RBC and the LR identification when we analyze the data simulated from the standard model. However, for demonstrational purposes, we also recover the IRFs under the SR identification with data simulated from the standard model, even if it is not applicable. After, to assess the estimation bias, we compare the mean of the estimated impulse responses with true

[25] Thus, the contemporaneous impulse responses are $\xi_z\Delta$ and 0 in the standard and recursive models, correspondingly. The responses in period $t \geq 1$ can be computed as $\gamma_k^{t-1}\xi_z\gamma_z\Delta$ for the standard model and $\left(\tilde{\xi}_z + \xi_k\tilde{\gamma}_z\right)\Delta$ for $t=1$, $\xi_k\gamma_k^{t-2}\left(\gamma_k\tilde{\gamma}_z + \tilde{\gamma}_z'\right)\Delta$ for $t \geq 2$ for the recursive.

[26] This are the same number of observations as contained in the datasets of Christiano et al. (2003, 2006), Francis and Ramey (2005) or Gali and Rabanal (2004).

responses from the corresponding RBC model. As well we compute the 95% confidence intervals for the estimated mean of responses.

As in Christiano et al. (2006), we assess the accuracy of the confidence intervals by computing coverage rates. The coverage rate is the fraction of times that the confidence intervals contain the true value of interest. If the 95% confidence intervals were perfectly accurate, the coverage rate would be 95%. To compute coverage rates, we simulated 100 datasets from both types of RBC models: the standard and the recursive. For each dataset we estimate the impulse responses under the two sets of identification assumptions and we compute confidence intervals by non-parametric bootstrap in frequency domain. To do so we make 200 bootstrap replication for each estimated model[27]. The confidence interval is defined as mean plus and minus 1.96 standard deviations across 200 bootstrapped impulse responses.

4.4 Results of the Sims procedure

Figure 4 depicts the results of the Sims procedure. Figure 4.a plots the mean of 500 estimated VARFIMA impulse responses of hours to a positive technology shock and their 95% confidence intervals, using the data generated by the recursive version of the RBC model. Figure 4.b shows analogous results for the standard RBC model. The first and second columns of each figure draw the response of hours uncovered with the SR (first column) and the LR (second column) identification schemes. In both cases we include the true impulse responses from the corresponding RBC model. To make our results comparable with the ones reported in the literature, we also plot the responses from the two standard VAR(4) with I(0) and I(1) hours.

As can be seen in the Figure 4.a, the mean of the VARFIMA responses uncovered with SR identification using data from the recursive RBC are positive and statistically significant in the short run. They follow zero contemporaneous response resulting from the SR identification restriction. The mean of the estimated responses is very close to the responses from the theoretical model. The results are similar to the ones achieved with VAR assuming that hours are I(0). Nevertheless, the responses recovered from VARFIMA seem to have been estimated more precisely with smaller

[27] The results of this procedure should be evaluated with caution since 100 simulated datasets and 200 bootstrap replications are few to accurately assess precision. Unfortunately, even these relatively few numbers already imply 20000 estimations of the VARFIMA model, which is computationally much more complex than the estimation of a standard VAR.

confidence intervals. Notice that VARFIMA responses are statistically different from zero up to ninth lag whereas the responses from VAR are not statistically significant after the third lag. The mean of VAR IRFs assuming that hours are I(1) are as well positive and statistically significant for the first three lags, but the estimated mean is further away from the real responses from the model and presents slower decay. This result is not surprising since assuming that hours are non-stationary, we make a misspecification with respect to RBC model, where hours are stationary. It is also important to notice, that, unlike VAR responses, the estimated VARFIMA responses with real data (Figure 2) are very similar in magnitude to the theoretical ones and to the ones recovered from the model.

Give that the recursive version of the model also satisfies the LR identification, we can apply this assumption to uncover the responses of hours from this model too. Results may be found in the second column of the Figure 4.a. The first result that pops out is that the mean of the VARFIMA responses is again very close to the real one from the recursive RBC model. The second is that there is a huge increase in the wideness of the confidence intervals: they are three times wider than ones resulting from the SR identification and always include zero. Thus, the sampling uncertainty associated to the LR identification is substantially larger than in the SR case. Similar conclusion is drawn from the estimation of a VAR with I(0) hours. However, if the responses are estimated by a VAR with I(1) hours, the mean of estimated responses is negative, statistically not different from zero and the responses from the RBC model are not even inside the confidence intervals. Note than in this last case the confidence intervals for the LR identification shrink. So, the mean of the estimated responses is not significant not because the sample uncertainty is higher, but because the mean is very close to zero.

The difference between the first and the second column of the Figure 4.a is especially interesting. Since we use the same artificial datasets and the same estimation procedure, the only difference in the estimated mean responses of hours plotted is the identification assumption used to recover them. Thus the differences between the results of the first and the second columns for the third row evidence the unsuitability of the I(1) assumption on hours for data generated from the model. Furthermore, for the VARFIMA case (first row), the assumption on the order of integration of hours should not exert any influence since it is relaxed. Thus the only reason of the increase in the sampling uncertainty is the use of the LR identification assumption.

To proceed with the evaluation of the LR identification we employ data simulated by the standard (non-recursive) RBC model. As commented before, the non-recursive RBC satisfies only the LR identification assumption. However, we also recover the impulse responses applying SR as an illustrative counterexample.

The mean responses estimated under the LR restrictions from the data simulated with the standard RBC model and their 95% confidence intervals are plotted in the Figure 4.b, second column. The mean of the estimated VARFIMA responses is very close to the real responses from the standard RBC model. The confidence intervals are wide and include zero, indicating high sampling uncertainty, as in the recursive version. The results from VAR with I(0) hours are again similar to the ones from VARFIMA. Results obtained from VAR with I(1) hours once more confirm the inappropriateness of the I(1) assumption for the simulated data: the mean response of hours is negative and the real responses are outside the confidence intervals.

For demonstrational purposes the first column of Figure 4.b presents the mean response of hours recovered with the SR assumption, even if it is not satisfied by the DGP. It is clear from the results that the SR is not applicable in this case: the mean of the estimated responses is very close to zero and the responses from the standard RBC model are not inside the confidence intervals of any model.

To check the precision of the VARFIMA confidence intervals, we compute their coverage rates. The Figure 5 (left panel) depicts the coverage rates for the confidence intervals using data generated by the recursive RBC, for both SR and LR identification assumptions. The coverage rate at lag zero when the SR identification is employed is one due to the identification assumption. The figure exhibit two striking features. First, both coverage rates are high. With very high probability, the VARFIMA based confidence intervals include the true value of the impulse response coefficients no matter the identification scheme chosen by the econometrician. Second, the coverage rate is higher for SR identification: if the true DGP is a recursive RBC, the SR identification is expected to give slightly better results. The right panel of Figure 5 presents the coverage rates for the confidence intervals computed with VARFIMA using data generated by the standard RBC model. If the LR identification is applied, the coverage rates are very high and relatively close to 0.95. Contrary, if we apply SR identification (not satisfied in the standard RBC model), the coverage rates are very low: it is very improbable that the confidence intervals based on this procedure contain the true value of the impulse response coefficients. Also, the confidence intervals always contain zero. This is important since it means that the responses uncovered with the SR scheme are never statistically significant when the SR is not valid. Thus, it seems that data tells whether SR can be used or not.

In general, based on the results of the Sims procedure, an econometrician, in general, would not be misled in inference by using a standard practice to compute the confidence intervals for impulse responses in VARFIMA, no matter the identification scheme she applies.

Summarizing, we do not find the evidence of the bias in the impulse responses estimated with VARFIMA under the SR and LR identification assumptions. The SR identification gives very good results. Application of the LR identification leads to a substantial increase in the sample uncertainty. However, the confidence intervals for the impulse responses of hours worked to a technology shock are found to be precise enough to make inference. The results are in line with results of Christiano et al. (2006) who studied performance of the short-run and long-run identification schemes in a structural VAR with I(0) hours worked. It is not surprising since both models are different types of approximation to the infinite VAR representation of the RBC model. As argued in Christiano et al. (2006), the main reason of the increase of the sampling uncertainty when LR identification is applied are the difficulties in approximating the infinite sum of VAR coefficients by a finite VAR. Unfortunately, application of fractional models does not help to solve this problem. Although fractional models do not restrict VAR to have a finite number of lags, the sum of autoregressive coefficients in the infinite VAR representation of VARFIMA is equal to zero or infinity depending on the coefficients of fractional integration of variables. What we do show in this section is that the VARFIMA process is applicable if the true DGP is an RBC model and that the impulse responses recovered with real data are comparable with the theoretical ones. This is very important because it is clear that the hours generated by the RBC model are much less persistent than the real data. The evidence that real hours are fractionally integrated, possibly non-stationary and mean reverting is very strong and not influenced by the definition of hours[28]. Hence the I(0) assumption appears to be too much restrictive when assessing the responses with real data.

[28] The most common explanation for the existence of fractional integration is the strong aggregation produced in real data (see e.g. Granger (1980)).

5. Conclusion

In this paper we relax the arguable assumption on the order of integration of hours worked when estimating the impulse responses of the hours to a (positive) technology shock. This assumption has been the subject of a debate in the literature for last ten years. Christiano et al. (2003) show that the response of hours is positive if hours are assumed to be integrated of order zero but negative otherwise.

To do so, we use VARFIMA model to estimate the order of fractional integration of hours at zero frequency together other parameters of the model. We estimate impulse responses under SR and LR identification assumptions. To assess the plausibility of the empirical model and the identification assumptions to identify the technology shock, we apply approach described in Sims (1989). We use parameterized RBC model (standard and recursive) as a data generating mechanisms to simulate a set of artificial series for hours and productivity. Thereafter, we estimate the simulated data with the VARFIMA model and recover the impulse responses under the same set of assumptions that are satisfied in the DGP. These impulse responses are compared with the ones from the theoretical RBC model.

According to our results, hours worked are found to be fractionally integrated, possibly non-stationary mean reverting. The order of integration of productivity is always not statistically different from one. Once the assumption of the order of integration is relaxed, the sign and the magnitude of the estimated responses of hours to a technology shock from the real data seem to depend on the identification scheme exclusively. This result is robust to changes in the specification and not influenced by movements at high frequencies or by the choice of the sample.

In the real data analysis, SR identification produces positive and statistically significant impulse responses that are very stable across datasets. It is interesting to note that the estimated from the data IRFs have not only the same sign as the theoretical IRFs from the recursive RBC model but also they are very close in magnitude. According to the Sims procedure, SR identification performs remarkably well: there is no evidence of bias in the estimated responses and confidence intervals seem to be enough precise to make inference.

The LR identification is still appropriate to back up the responses of hours in the VARFIMA model however, the sample uncertainty increases dramatically when it is applied. Nevertheless, the coverage rates for confidence intervals confirm that with very high probability VARFIMA based confidence intervals include the true value of the impulse response coefficients. With real data, the LR scheme produces negative impulse responses yet with wide confidence intervals. In fact, if productivity and hours are defined only for the non-agricultural sector, the responses become not statistically significant. Moreover, the results on the significance seem to be also sample-dependent. In the sub-sample started from 1959:1 the responses are not significant already in one additional dataset. Thus, significance of the responses uncovered with LR identification is both data and sample dependent.

In this study we were primary concerned with the apparent contradiction between the real data analysis and the standard RBC model. Thus, in order to ensure that empirical VARFIMA responses recovered from the real data can be compared with those from the RBC, we have employed this model for the simulation study. However, it would be interesting to assess if the VARFIMA responses can be also compared with those arising from models that cause a drop in hours up front. Specially, if one considers that the RBC is not a valid model to explain the behavior of hours. To address these issues, additional studies featuring, for example, RBC model augmented with sticky prices and wages, and a monetary policy rule not responding aggressively to the output gap are required. We leave these interesting questions open for future research.

References

Berkowitz, J. and F.X. Diebold (1998) Bootstrapping multivariate spectra. *The review of economics and statistics* 80(4), 664-666.

Blanchard, O.J. and D. Quah (1989) The dynamic effects of aggregate demand and supply disturbances, *The American Economic Review* 79(4), 655-673.

Boes, D.C., Davis, R.A. and S.N. Gupta (1989) Parameter estimation in low order fractionally differenced ARMA processes, *Stochastic Hydrology and Hydraulics* 3, 97-110.

Canova, F., Lopez-Salido, D., and C. Michelacci (2007) The labor market effects of technology shocks. *Documentos de Trabajo No. 0719*. Banco de España.

Chari, V.V., Kehoe, P.J., and E.R. McGrattan (2008) Are structural VARs with long-run restrictions useful in developing Business Cycle Theory? *Journal of Monetary Economics* 55(8), 1337-1352.

Christiano, L. J., Eichenbaum, M., and R. Vigfusson (2003) What happens after a technology shock? FRB International Finance Discussion Paper No. 768. Board of Governors of the Federal Reserve System.

Christiano, L. J., Eichenbaum, M., and R. Vigfusson (2006) Assessing structural VARs. International Finance Discussion Paper 866. Board of Governors of the Federal Reserve System.

Erceg, C.J., Gust C., and L. Guerrieri (2005) Can long-run restrictions identify technology shocks? *Journal of the European Economic Association* 3 (6), 1237-1278.

Faust, J. and E. Leeper (1997) When do long-run identifying restrictions give reliable results? *Journal of Business and Economic Statistics* 15(3), 345-353.

Fernandez-Villaverde, J., Rubio-Ramirez, J.F., Sargrnt, T.J, and M.W. Watson (2007) A, B, C's (and D's) for understanding VARs. *American Economic Review* 97 (3), 1021-1026.

Fernald, J.G. (2007) Trend breaks, long-run restrictions, and contractionary technology improvements. *Journal of Monetary Economics* 54 (8), 2467-2485

Francis, N., Owyang, M.T., and J.E. Roush (2005) A flexible finite-horizon identification of technology shocks, International Finance Discussion Paper Number 832. Board of Governors of the Federal Reserve System.

Francis, N. and V.A. Ramey (2005) Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited. *Journal of Monetary Economics* 52, 1379-1399.

Gali G. (1999) Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *The American Economic Review* 89 (1), 249-271.

Gali, J. and P. Rabanal (2004) Technology shock and aggregate fluctuations: How well does the RBC model fit post War US data? CEPR Discussion Paper No. 4522.

Gil-Alana, L.A., A. Moreno (2009) Technology shocks and hours worked: A fractional integration perspective. *Macroeconomic Dynamics* 13, 580-604.

Granger, C. W. J. (1980) Long memory relationships and the aggregation of dynamic models. *Journal of Econometrics* 14, 227-38.

Hassler U., (1994) (Mis)specification of long memory in seasonal time series. *Journal of Time Series Analysis* 14 (1), 19-30.

Hosoya, Y. (1996) The quasi-likelihood approach to statistical inference on multiple time-series with long-range dependence. *Journal of Econometrics* 73, 217-236.

Gil-Alana, L.A. and P.M. Robinson (2001) Testing of seasonal fractional integration in the UK and Japanese consumption and income. *Journal of Applied Econometrics* 16, 95-114

Lovcha, Y, Perez-Laborda, A and L.A. Gil-Alana (2012) On the Invertibility of seasonal series adjusted by TRAMO-Seats. Typescript.

Lovcha, Y and A. Perez-Laborda (2010) Seasonal misspecification in long memory processes: a simulation study, Typescript.

Porter-Hudak, S., (1990) An application of the seasonal fractionally differenced model to the monetary aggregates. *Journal of the American Statistical Association* 85, 338-344.

Robinson, P.M. (1994) Semi-parametric analysis of long-memory time series. *The Annals of Statistics* 22 (1), 515-539.

Ray, B.K. (1991) Fractionally differenced ARMA processes: seasonality and forecasting issues, *PhD thesis*, Graduate School of Arts and Science, Columbia University, USA.

Ray, B.K. (1993) Long-range forecasting of IBM product revenues using a seasonal fractionally differenced ARMA model. *International Journal of Forecasting* 9, 255-269.

Sims, C. (1972) The role of approximate prior restrictions in distributed lag estimation. *Journal of the American Statistical Association* 67 (337), 169-175.

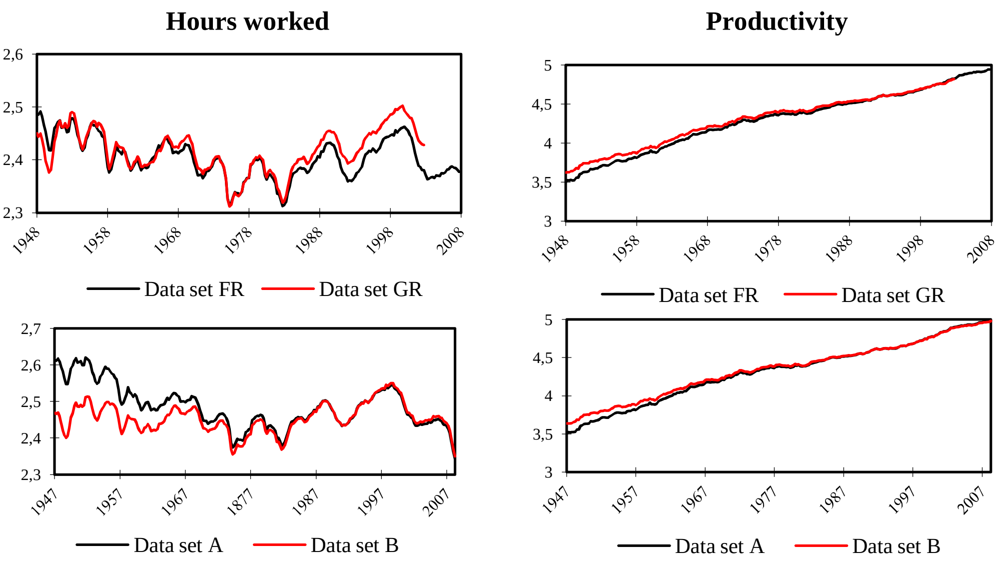
Sims, C. (1989), Models and their uses. *American Journal of Agricultural Economics* 71, 489-494.

Tschernig, R., Weber, E., and R. Weigand, (2010) Long-run identification in fractionally integrated system. University of Regensburg Working Papers in Business, Economics and Management Information Systems, 447.

Uhlig, H. (2004) Do technology shocks lead to a fall in total hours worked? *Journal of the European Economic Association* 2(2-3), 361-371.

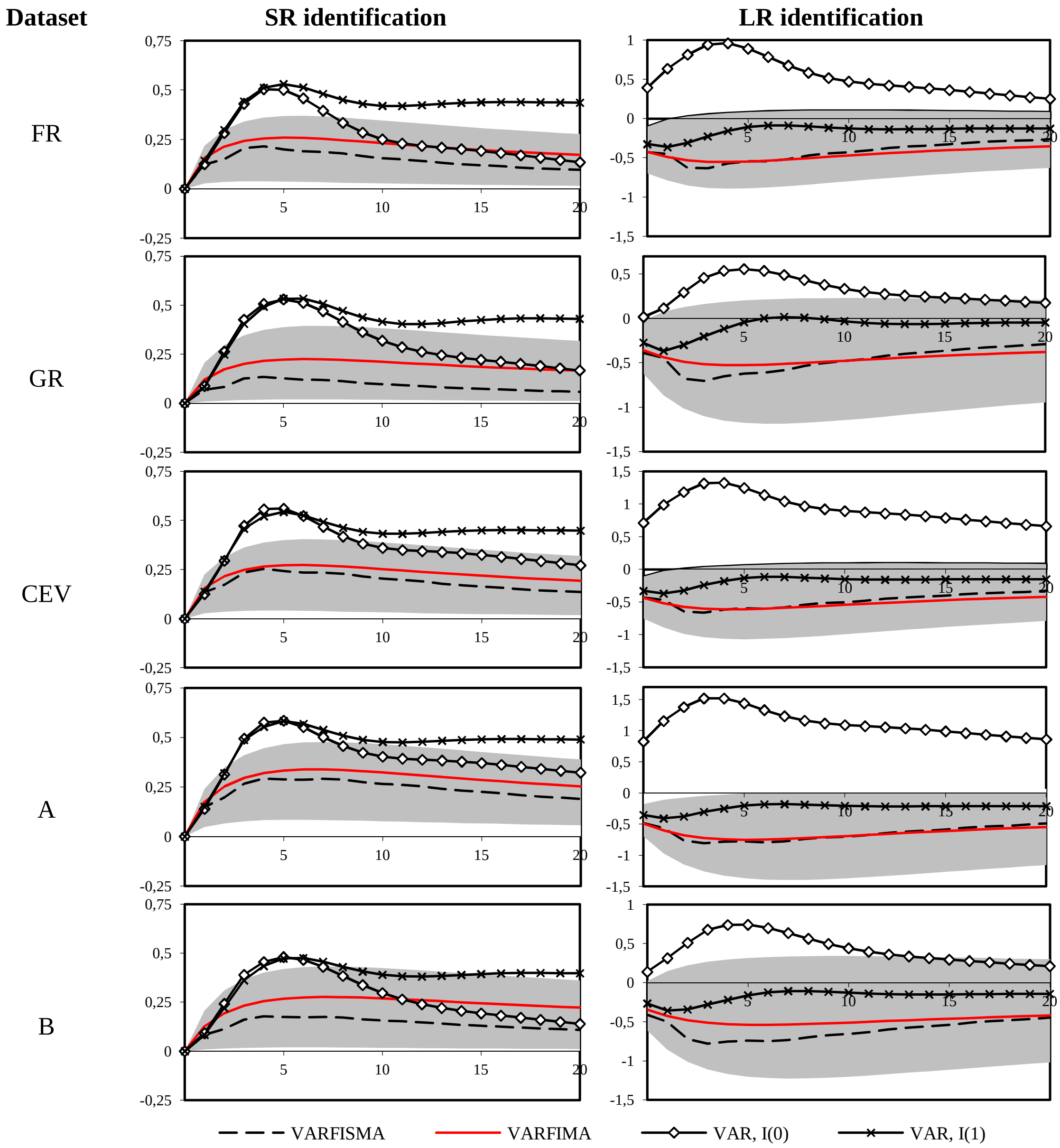
Graphs and Tables

Figure 1. Hours worked and Productivity in different data sets.



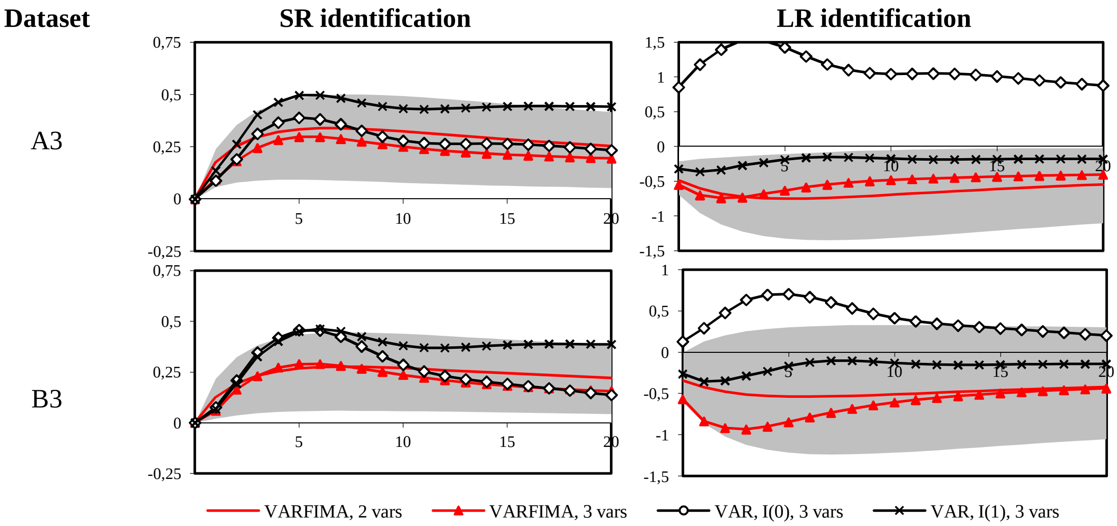
Notes: (a) FR corresponds to the data set of Francis and Ramey (2004), GR - Gali and Rabanal (2004), data sets A and B are extended versions of the data sets of Christiano, Eichenbaum and Vigfusson (2003) and Gali (1999). (b) All variables are transformed to the logarithm form.

Figure 2. Impulse responses of hours worked to a positive technology shock estimated from two-variable datasets.



Notes: (a) in grey - 95% confidence intervals computed with non-parametric bootstrap for IRFs from VARFIMA model with productivity restricted to be I(1). (b) Left, SR identification; Right, LR identification. (c) FR corresponds to the data set of Francis and Ramey (2004), GR - Gali and Rabanal (2004), CEV - Christiano, Eichenbaum and Vigfusson (2003). Data sets A and B are extended versions of the data sets of Christiano, Eichenbaum and Vigfusson (2003)and Gali (1999).

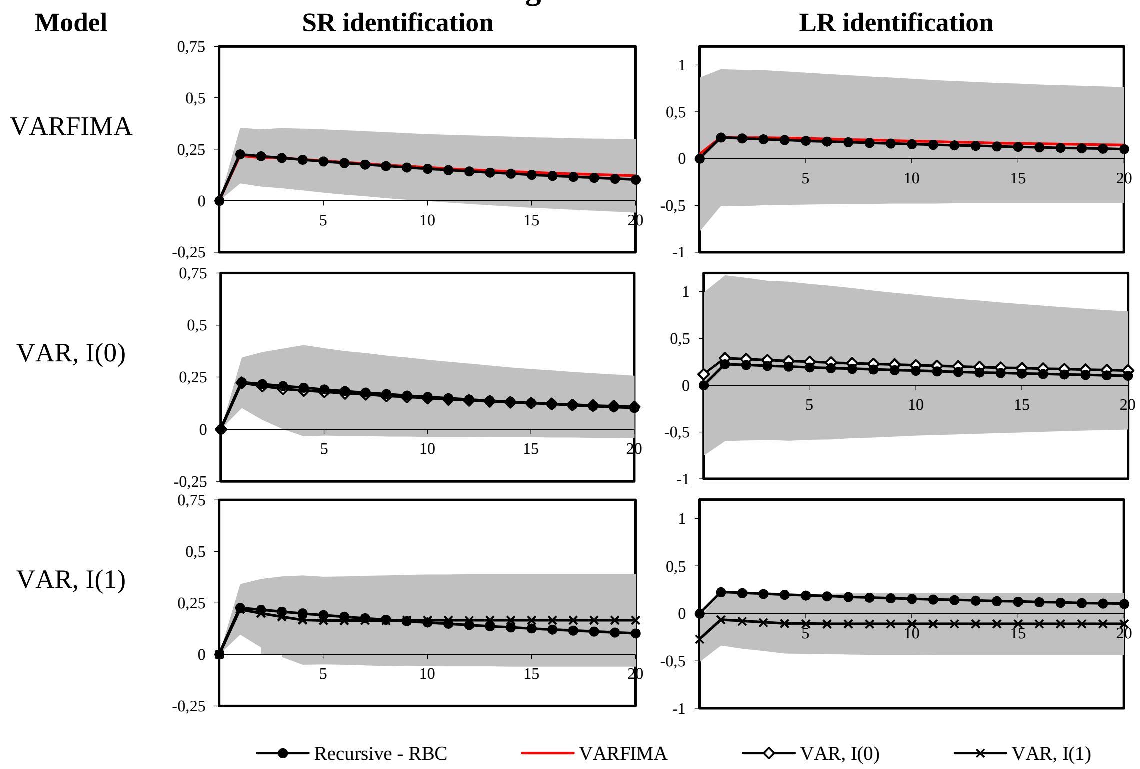
Figure 3. Impulse responses of hours worked to a positive technology shock estimated from three-variable datasets (including investment-output ratio).



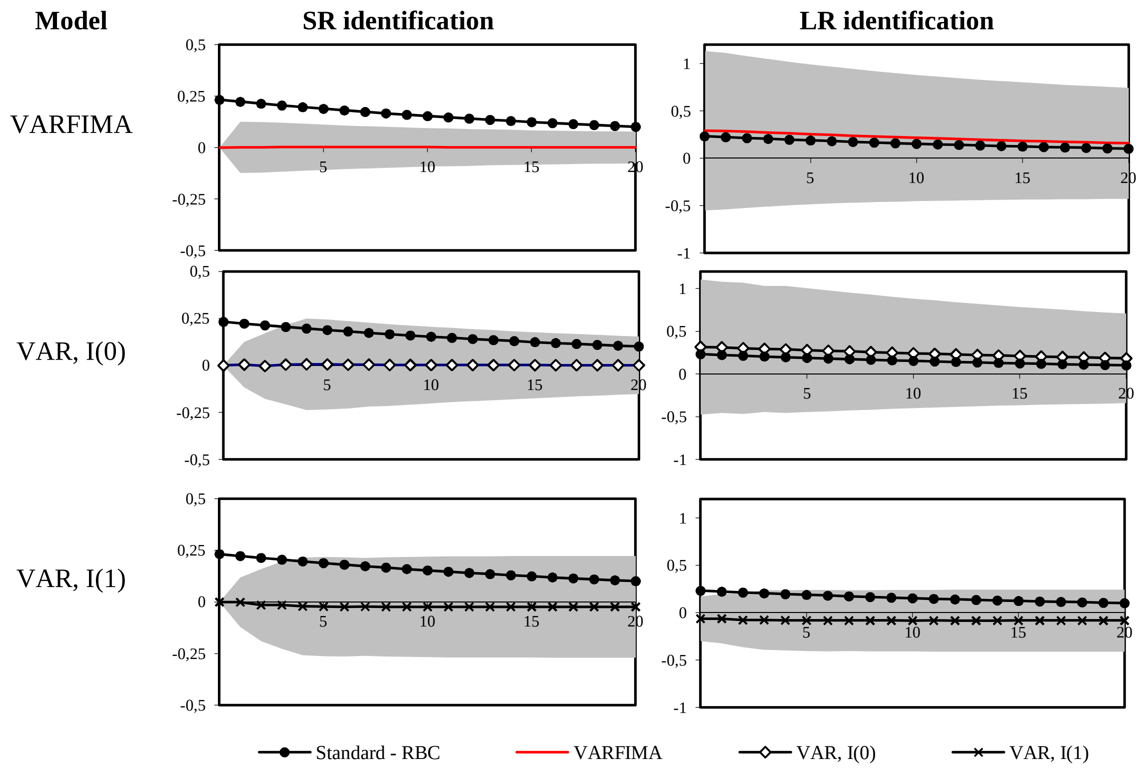
Notes: (a) in grey - 95% confidence intervals computed with non-parametric bootstrap for IRFs from VARFIMA model with productivity restricted to be I(1). (b) Left, SR identification; Right, LR identification. . (c) Data sets A3 and B3 are extended versions of the data sets of Christiano, Eichenbaum and Vigfusson (2003) and Gali (1999) including investment-output ratio as third variable.

Figure 4. Simulations results: mean of estimated impulse responses of hours worked to a positive technology shock; Data generated from: Recursive RBC (panel up), Standard RBC (panel down).

Fig 4.a Recursive RBC

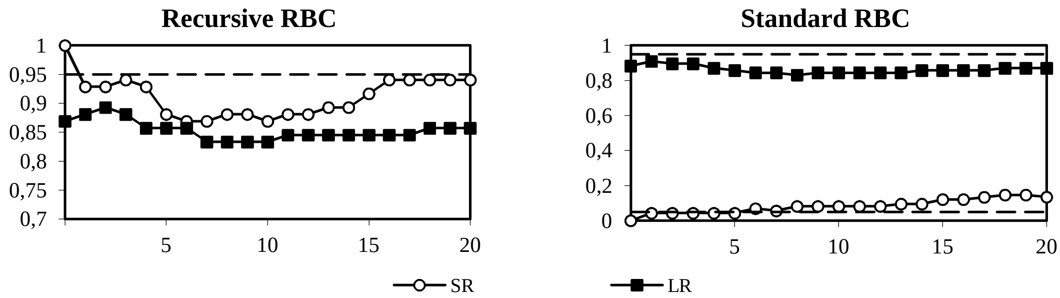


Standard RBC



Notes: (a) in grey - 95% confidence intervals. (b) SR identification is not satisfied in the standard RBC model.

Figure 5. Coverage rates for confidence intervals from VARFIMA computed with short-run (SR) and long-run (LR) identification; Data generated from: Recursive RBC (left), Standard RBC (right).



Notes: (a) The coverage rate is the fraction of times that the confidence intervals contain the true value of interest. If the 95% confidence intervals were perfectly accurate, the coverage rate would be 95%. (b) SR identification assumptions are not satisfied in the standard RBC model.

Table 1. Unit root testing of Hours, p-values of the tests.

	dfARDTest	dfARTest	dfTSTest	ppARDTest	ppARTest	ppTSTest
FR	0.0054*	0.4701	0.0194*	0.1336	0.3427	0.4357
GR	0.0094*	0.6077	0.0418*	0.4000	0.5774	0.6858
CEV	0.1403	0.5222	0.3996	0.2622	0.3959	0.5823
A	0.3007	0.9219	0.2882	0.7483	0.9852	0.8799
B	0.0195*	0.8129	0.0989	0.7763	0.8849	0.9668

Notes: (a) H0: the true underlying process is a unit root process with or without drift; (b) dfARDTest - Augmented Dickey-Fuller unit root test based on AR model; dfARTest - Augmented Dickey-Fuller unit root test based on zero drift AR model; dfTSTest - Augmented Dickey-Fuller unit root test based on trend stationary AR model; ppARDTest – Phillips Perron unit root test based on AR(1) model with drift; ppARTest - Phillips-Perron unit root test based on zero drift AR(1) model; ppTSTest - Phillips-Perron unit root test based on trend stationary AR(1) model; dfARDTest, dfARTest, dfTSTest are corrected for serial correlation of residuals in the underlying OLS; (c) * H0 is rejected at 10% significance level.

Table 3. Estimation results.

Estimates	FR				GR				CEV				A				B			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
d_{10}	0.660 (0.087)	0.635 (0.084)	0.481 (0.106)	0.460 (0.099)	0.693 (0.093)	0.682 (0.093)	0.458 (0.100)	0.441 (0.096)	0.689 (0.093)	0.653 (0.082)	0.561 (0.118)	0.506 (0.103)	0.678 (0.089)	0.655 (0.080)	0.531 (0.113)	0.508 (0.103)	0.709 (0.091)	0.712 (0.088)	0.501 (0.103)	0.506 (0.090)
d_{11}	-	-	-0.138 (0.076)	-0.151 (0.075)	-	-	-0.231 (0.081)	-0.243 (0.080)	-	-	-0.116 (0.079)	-0.145 (0.071)	-	-	-0.113 (0.075)	-0.138 (0.074)	-	-	-0.192 (0.069)	-0.204 (0.067)
d_{12}	-	-	-0.219 (0.076)	-0.228 (0.075)	-	-	-0.286 (0.077)	-0.318 (0.077)	-	-	-0.164 (0.083)	-0.197 (0.079)	-	-	-0.175 (0.077)	-0.162 (0.075)	-	-	-0.239 (0.068)	-0.217 (0.069)
d_{20}	1.078 (0.080)	-	0.979 (0.083)	-	1.070 (0.083)	-	0.994 (0.094)	-	0.914 (0.085)	-	0.998 (0.088)	-	1.078 (0.08)	-	0.963 (0.086)	-	1.036 (0.087)	-	0.945 (0.097)	-
d_{21}	-	-	-0.085 (0.067)	-	-	-	-0.020 (0.067)	-	-	-	-0.055 (0.074)	-	-	-	-0.100 (0.071)	-	-	-	-0.026 (0.067)	-
d_{22}	-	-	0.126 (0.116)	-	-	-	0.153 (0.098)	-	-	-	0.201 (0.121)	-	-	-	0.143 (0.111)	-	-	-	0.114 (0.102)	-
F_{11}	0.793 (0.066)	0.807 (0.063)	0.817 (0.070)	0.822 (0.065)	0.800 (0.064)	0.804 (0.065)	0.820 (0.066)	0.818 (0.064)	0.778 (0.075)	0.816 (0.064)	0.801 (0.085)	0.833 (0.070)	0.816 (0.070)	0.843 (0.062)	0.852 (0.074)	0.871 (0.063)	0.811 (0.064)	0.820 (0.062)	0.839 (0.065)	0.850 (0.057)
F_{12}	0.174 (0.056)	0.183 (0.055)	0.164 (0.057)	0.144 (0.056)	0.128 (0.059)	0.140 (0.057)	0.105 (0.061)	0.082 (0.057)	0.175 (0.060)	0.182 (0.056)	0.171 (0.061)	0.159 (0.057)	0.187 (0.057)	0.206 (0.056)	0.174 (0.058)	0.175 (0.057)	0.147 (0.056)	0.1531 (0.055)	0.101 (0.058)	0.102 (0.057)
F_{21}	-0.169 (0.049)	-0.167 (0.047)	-0.160 (0.050)	-0.163 (0.049)	-0.173 (0.051)	-0.167 (0.049)	-0.183 (0.051)	-0.187 (0.057)	-0.160 (0.052)	-0.163 (0.046)	-0.150 (0.054)	-0.150 (0.049)	-0.161 (0.046)	-0.162 (0.043)	-0.148 (0.047)	-0.136 (0.044)	-0.144 (0.046)	-0.148 (0.044)	-0.148 (0.046)	-0.159 (0.044)
F_{22}	-0.147 (0.100)	-0.075 (0.065)	0.091 (0.151)	-0.054 (0.065)	-0.139 (0.105)	-0.053 (0.067)	0.121 (0.159)	-0.046 (0.067)	-0.158 (0.105)	-0.075 (0.064)	0.154 (0.161)	-0.050 (0.065)	-0.143 (0.100)	-0.053 (0.064)	0.126 (0.151)	-0.049 (0.064)	-0.049 (0.109)	-0.008 (0.063)	0.183 (0.164)	-0.005 (0.063)
σ_1^2	0.536 (0.049)	0.536 (0.034)	0.508 (0.047)	0.512 (0.033)	0.562 (0.054)	0.560 (0.036)	0.515 (0.050)	0.511 (0.035)	0.595 (0.058)	0.548 (0.034)	0.566 (0.056)	0.525 (0.034)	0.563 (0.051)	0.559 (0.034)	0.538 (0.050)	0.548 (0.034)	0.527 (0.048)	0.524 (0.033)	0.490 (0.045)	0.504 (0.033)
σ_2^2	0.736 (0.068)	0.731 (0.039)	0.720 (0.067)	0.724 (0.039)	0.784 (0.076)	0.776 (0.043)	0.736 (0.072)	0.778 (0.042)	0.782 (0.076)	0.730 (0.039)	0.775 (0.077)	0.719 (0.039)	0.725 (0.066)	0.723 (0.039)	0.701 (0.065)	0.720 (0.039)	0.718 (0.065)	0.707 (0.038)	0.684 (0.063)	0.687 (0.038)
σ_{12}	0.006 (0.041)	0.014 (0.041)	-0.001 (0.040)	0.014 (0.040)	0.113 (0.046)	0.123 (0.046)	0.121 (0.046)	0.131 (0.043)	0.025 (0.047)	0.020 (0.041)	0.026 (0.047)	0.018 (0.041)	0.025 (0.041)	0.030 (0.041)	0.021 (0.041)	0.029 (0.041)	0.099 (0.040)	0.108 (0.040)	0.116 (0.039)	0.129 (0.039)

Notes: 1 – the data is fit with VARFIMA; 2 – the data is fit with VARFIMA restricting productivity to be I(1); 3 – the data is fit with VARFISMA model; 4 - the data is fit with VARFISMA, restricting productivity to be I(1) process at zero frequency and I(0) at seasonal frequencies; standard errors are presented in parenthesis; In all models, the order of autoregression is chosen by Schwarz information criteria.