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and inflation dynamics

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**DEPARTAMENT D'ECONOMIA – CREIP**  
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# A FRACTIONALLY INTEGRATED APPROACH TO MONETARY POLICY AND INFLATION DYNAMICS

Yuliya Lovcha<sup>\*</sup>      Alejandro Perez-Laborda<sup>§</sup>

## Abstract:

This paper relaxes the standard  $I(0)$  and  $I(1)$  assumptions typically stated in the monetary VAR literature by considering a richer framework that encompasses the previous two processes as well as other fractionally integrated possibilities. First, a time-varying multivariate spectrum is estimated for post WWII US data. Then, a structural fractionally integrated VAR (VARFIMA) is fitted to each of the resulting time dependent spectra. In this way, both the coefficients of the VAR and the innovation variances are allowed to evolve freely. The model is employed to analyze inflation persistence and to evaluate the stance of US monetary policy. Our findings indicate a strong decline in the innovation variances during the great disinflation, consistent with the view that the good performance of the economy during the 80's and 90's is in part a tale of good luck. However, we also find evidence of a decline in inflation persistence together with a stronger monetary response to inflation during the same period. This last result suggests that the Fed may still play a role in accounting for the observed differences in the US inflation history. Finally, we conclude that previous evidence against drifting coefficients could be an artifact of parameter restriction towards the stationary region.

**Keywords:** monetary policy, inflation persistence, fractional integration, time-varying coefficients, VARFIMA

**JEL Classification:** E52, C32

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## 1. Introduction

During the last forty years, the U.S. economy has been characterized by markedly different episodes concerning both the level and the volatility of inflation. As can be seen in Figure 1, the U.S. annual average rate of inflation declined from a 4.5% in the period 1960-1984 to 3% in 1985-2009. Also its standard deviation decreased by one third between these two periods. This has led to research trying to assess the role of the Fed in accounting for these observed differences. Whether the high inflation episode had been a consequence of bad policy or rather, the result of bad luck is still controversial in the literature. While some authors assert that monetary policy has been conducted more efficiently starting from Volker's chairmanship (see e.g. Boivin and Gianonni (2006), Clarida et al. (2000), Cogley and Sargent (2002), or Lubick and Shorftheide (2007)) other's found small or null evidence of drastic changes in the monetary policy from about fifteen years prior to Volker (main references include Bernanke and Mihov (1998), Canova and Gambetti (2009), Primiceri (2005) or Sims and Zha (2006). According to the last group of authors, the main factor driving the differences between the two periods was a reduction of the volatility of the exogenous shocks. For instance, Sims and Zha (2006) do not find any evidence of coefficient drifting once time variability in the structural disturbances is taken into account.

This paper investigates the possible causes of the poor economic performance of the 70 and early 80's, and the role of the Fed in the observed changes. The objective here is to relax the strong assumptions on the persistence of inflation typically made in previous literature. Most of it has relied on the analysis of vector autorregresion (VAR). Given that the nature of the question is

fully dynamic, the models have to be estimated at different sample periods or the parameters must be able to change. In fact, time-varying VAR's have recently received a lot of attention in the literature<sup>1</sup>. Despite their different conclusions, previous research has restricted the VAR parameter space to deliver stationarity results. The stationarity assumption contrasts to a large stream of different literature which agrees that inflation is better characterized by an  $I(1)$  process (see e.g. Benati and Surico (2008), Cogley, Primiceri and Sargent (2010), Pivetta and Reis (2007) or Stock and Watson (2007))<sup>2</sup>. Furthermore, by restricting the VAR parameter space, one is imposing not only stationarity, but also short memory ( $I(0)$ ). The distinction between  $I(0)$  and  $I(1)$  cases is not minor. Although both formulations can deliver similar short term predictions if appropriate parameters are chosen, the medium and long run implications (frequently the object of interest in macroeconomics) are drastically different. While the autocorrelation function of  $I(0)$  process show exponential decay with the effect of the shocks dying in the short run,  $I(1)$  process are characterized by a flat autocorrelation function revealing that shocks are permanent. Besides, if the VAR parameters are forced towards the stationary region and inflation is not  $I(0)$ , one may question up to which extent the coefficient drifting found in previous literature may be underestimated in front of changes in the variance.

To address these issues this paper considers a wider statistical framework that encompasses both  $I(0)$  and  $I(1)$  assumptions as well as other fractionally

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<sup>1</sup> See among others the work of Boivin and Gianonni (2006), Canova and Gambetti (2009), Cogley and Sargent (2002, 2005a) or Primiceri (2005).

<sup>2</sup> Pivetta and Reis (2007) calculate that imposing such a restriction leads to more than 40% of parameter rejections. From the other side, evidence on unit root tests is mixed and strongly period specific. However several authors were not able to reject the unit root null for long period of time (see e.g. Murray et al. (2008) and references there in).

integrated possibilities. First, a time varying spectrum is estimated for post WWII US data and then a fractionally integrated (FI) vector autoregressive model (VARFIMA) is fitted in the frequency domain to each of the resultant time dependent spectrums. Thus, the entire model's parameters change with the time varying spectrum obtaining smooth parameter transitions without the need of parametric specification of the laws of motion. This is also an advantage with respect to the standard time varying VAR literature, where a dynamic specification must be assumed at the outset.

Fractional integration account for situations where the  $I(0)$  and  $I(1)$  assumptions are too restrictive, allowing the effect of the shocks not to be permanent but decaying at a rate lower than exponential, which is the actual behavior of many macroeconomic time series<sup>3</sup> (see e.g. Henry and Zaffaroni (2002) for significant references). In fact, fractionally integrated models have been successfully employed for modeling inflation, typically by univariate approaches<sup>4</sup>. Most of this studies report evidence of a non-stationary but mean reverting behavior of inflation.

The statistical framework above is employed to asses the degree of stability of inflation persistence and to investigate changes in the way the monetary policy has been conducted by the Fed. Inflation persistence is a key factor in the design of monetary policy since it determines the monetary transmission mechanism and may affect Fed believes about the natural rate hypothesis. Our framework is particularly well suited to evaluate inflation

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<sup>3</sup> As shown in Gadea and Mayoral (2006), fractional integration may appear in inflation as the result of price aggregation over heterogeneous agents.

<sup>4</sup> Main references include Baillie et al. (1996), Baum et al. (1999), Bos et al. (2002), Hassler and Wolters (1995), Franses and Ooms (1997), Gil-Alana (2005) or Gadea and Mayoral (2006).

persistence, since it can be measured by the fractional integration parameter at zero frequency.

Our findings include the following. The variance of the shocks reduced drastically from the 80's, suggesting that a sizable part of the great performance in the economy during last 80's and 90's could be just a tale of good luck. However, even accounting for the (strong) reduction in the innovation variances, we find overwhelming evidence of a more active monetary policy towards inflation starting from Volker's and extending through first half of Greenspan's chairmanships. In line with the previous result, we find that persistence also have been falling from the early 80's, which most monetary models interpret as the result of a more vigorous attention to inflation on the part of the Fed. Nevertheless, inflation has remained very persistent during the whole period, characterized by a non-stationary but mean reverting behavior. This result questions the adequacy of the  $I(0)$  framework and suggests that previous evidence against drifting coefficients sometimes may be an artefact of parameter space restriction towards the stationary region. Overall, our conclusions are more similar to Cogley and Sargent (2005) or Fernandez-Villaverde et al. (2010).

The outline of the article is as follows: Section 2 describes the econometric framework. Empirical analysis may be found in section 3. Section 4 concludes.

## **2. Econometric Framework**

### **2.1 Estimation of the time varying cross-spectrum.**

As a first step of the analysis, we estimate a time-varying cross-spectrum of the variables. In the statistical literature, various methods have been proposed to estimate time-varying spectra, both parametric and non-parametric.

Examples of parametric methods for the estimation of time varying spectrum are given by Jansen et al. (1981), Kitagawa and Gersch (1996) and Davis et al. (2006). First, the time series is divided into blocks, and after the parameters of autoregressive models are estimated at each block. By substituting the estimated parameters in the spectral density of the corresponding autoregressive models, the spectrum of each block is obtained. Another solution, in principle, is to fit an autoregressive model with time-varying parameters to the data (see e.g. Cogley and Sargent (2002, 2005)). In any case, autoregressive models focus on fitting short run dynamics, and they are not expected to produce good estimates of the spectrum at low frequencies as required for fractionally integration estimation (see e.g. Christiano et al. (2006)).

From the other side, non-parametric estimation of the time-varying spectra is produced without parameterization of the spectrum. Thus, Cohen (1989) and Adak (1998) divide the time series into blocks and compute the sample spectrum for each block of observations. All time points in each block have the same estimate of the spectrum in this approach. The time-varying spectrum only evolves over the blocks. By using the smoothing spline ANOVA (SS-ANOVA) method (Gu and Wahba (1993) and Gu (2002)), the time series is smoothed over these initial block spectra. Smoothing is produced simultaneously over time and frequency. This procedure has been recently adopted by Koopman and Wong (2011) and is also the approach we follow in the present study.

Let  $x_{j,k}$  be the initial log-sample spectrum for frequency  $\lambda_k$  at time  $t_j$  (plus a constant). The log-sample spectrum is related to an unknown two dimensional function representing the true log-spectrum  $G(t_j, \lambda_k)$  as:

$$x_{j,k} = G(t_j, \lambda_k) + \zeta_{j,k}, \quad \zeta_{j,k} \sim N(0, \sigma_\zeta), \quad j = 1 \dots T, \quad k = 1 \dots M,$$

, where  $\zeta_{j,k}$  is an error term. Omitting interaction terms between  $\lambda_k$  and  $t_j$ , the unknown function  $G(t_j, \lambda_k)$  is modeled as a tensor product between two cubic spline functions  $G(t_j, \lambda_k) = G_1(t_j) + G_2(\lambda_k)$ , being  $G_1(t_j)$  and  $G_2(\lambda_k)$  the smoothed main effects over time and frequency. Defining a Bayesian stochastic model for  $G(t_j, \lambda_k)$  and the recursive Bayesian model, one can cast the SS-ANOVA into a State Space and obtain the estimates of  $G(t_j, \lambda_k)$  by employing the Kalman filter and smoother recursions (see Qin and Guo (2006) or Koopman and Wong (2011), for details).

Once the time varying cross spectral density is obtained, a VARFIMA model is fitted to each of the time dependent resultant cross-spectra estimates. The next sections review the VARFIMA model and its estimation procedure.

## 2.2 Model description and frequency domain estimation:

### 2.2.1. The VARFIMA model

Univariate ARFIMA models can be generalized to multivariate settings leading to the VARFIMA model. More specifically, the autoregressive VARFIMA model can be written as:

$$D(L)y_t = e_t \quad (1)$$

$$e_t = F(L)e_t + \varepsilon_t \quad (2)$$

where  $y_t$  is a  $N \times 1$  vector of variables for  $t = 1, \dots, T$ ;  $L$  is the backward shift operator; and  $D(L)$  is the diagonal  $N \times N$  matrix with the diagonal elements given by  $(1-L)^{d_n}$ ,  $n = 1, \dots, N$ . The *memory* parameter  $d_n$  is the parameter of fractional integration at zero frequency of the series  $y_t^n$ . As larger  $d_n$  is, more persistent variable is, and stronger policy actions are required to bring the variable to the targeted value. Stationarity requires  $d_n \in (-0.5, 0.5)$  which can always be achieved by taking a proper number of integer differences. Short

memory occurs when  $d_n=0$ , and the autocorrelations falling at exponential rate. If  $d_n \in (0,0.5)$  the process has long memory, and the autocorrelations show hyperbolic decay with the effect of the shocks taking more time to disappear than in the short memory case. The case  $d_n \in [0.5,1)$  is of a lot of interest in macroeconomics. The process is not stationary but presents mean reversion: the effect of the shocks eventually disappears. Shocks have permanent effect whenever  $d_n \geq 1$ .  $F(L)$  is the stationary autoregressive polynomial with  $p$  lags capturing short run dynamics. The  $N \times 1$  vector of errors  $\varepsilon_t$  is assumed to be  $N(0, \Omega)$ .

With a similar intention as here, the VARFIMA model has been recently employed in the business cycle literature assessing the response of hours worked to productivity shocks, although in a time invariant framework (Gil-Alana and Moreno (2008) and Lovcha (2009)).

### 2.2.2. The reduced and structural form of the VARFIMA model

The VARFIMA model defined as in (2) and (3) is a reduced form model. Substitution of (2) into (3) leads after arrangement to the reduced-form MA ( $\infty$ ) representation of  $y_t$ :

$$y_t = D^{-1}(L)(I_N - F(L))^{-1} \varepsilon_t \quad (3)$$

The structural model includes the contemporaneous relationships between variables, and it is given by:

$$AD(L)y_t = u_t \quad (4)$$

$$u_t = Q(L)u_t + \xi_t \quad (5)$$

where  $A$  is  $N \times N$  matrix of structural relationships. The vector of structural error terms  $\xi_t$  is assumed to be  $N(0, I_N)$ . Substitution of (7) into (8) and pre-multiplication of both sides by  $A^{-1}$  leads to:

$$\begin{aligned} D(L)y_t &= A^{-1}Q(L)AD(L)y_t + A^{-1}\xi_t \\ (I_N - A^{-1}Q(L)A)D(L)y_t &= A^{-1}\xi_t \end{aligned}$$

Implying:

$$y_t = D^{-1}(L)(I_N - A^{-1}Q(L)A)^{-1} A^{-1}\xi_t \quad (6)$$

which is the structural MA( $\infty$ ) representation of  $y_t$ .

To identify structural errors we apply Sims (1989) short-run assumptions by assuming that the matrix of contemporaneous relationships is lower-triangular<sup>5</sup>. It follows from (6) and (9) that  $A^{-1}\xi_t = \varepsilon_t$ . Once the variance-covariance matrix  $\hat{\Omega}$  of the reduced -form model errors is estimated, entries of the matrix of contemporaneous responses can be found.

### 2.2.3. Impulse responses

Substitution of  $A^{-1}\xi_t = \varepsilon_t$  into (4) leads to:

$$y_t = D^{-1}(L)(I_N - F(L))^{-1} A^{-1}\xi_t = \Lambda(L)\xi_t$$

Since the matrix  $D(L)$  is diagonal, its inverse is also diagonal, with elements given by  $D_n(L) = (1-L)^{-d_n}$ . The operator  $D_n(L)$  can also be defined by its infinite Taylor expansion:

$$(1-L)^{-d_n} = -\sum_{k=0}^{\infty} d_{n,k} L^k, \quad d_{n,k} = \frac{\Gamma(k-d_n)}{\Gamma(k+1)\Gamma(-d_n)}$$

where  $\Gamma(\cdot)$  denotes the gamma function, satisfying  $\Gamma(z+1) = z\Gamma(z)$ . The computation of the elements of the matrix  $(I_N - F(L))^{-1}$  is straightforward<sup>6</sup>. The matrix  $\Lambda(L)$  is  $N \times N$  and its elements are infinite MA polynomials which coefficients are impulse responses of variables to the structural shocks. Thus, the coefficients of the polynomial  $\Lambda_{lm}(L)$  are impulse responses of the variable  $l$  to a shock in  $m$ .

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<sup>5</sup>It is, the order of variables in the model matters; and a variable  $y_{n,t}$  is not contemporaneously influenced by any variable  $y_{n+l,t}$ ,  $l = n+1, \dots, N$ , but may be influenced by variables  $y_{l,t}$ ,  $l = 1, \dots, n$ .

<sup>6</sup>The Taylor expansion can be applied or the coefficients can be computed as in Hamilton (1994, p.260)

#### 2.2.4 The approximate maximum likelihood estimation of VARFIMA model

To estimate the process given by (2) and (3) we use approximate frequency domain maximum likelihood, also known as Whittle estimation (Boes et al. (1989)). The discussion of the multivariate version of the estimation procedure can be found in Hosoya (1996).

An approximate log-likelihood function of  $\theta$  based on  $y_t, t = 1, \dots, T$ , is given up to constant multiplication, by

$$\ln L(y, \theta) = - \sum_{j=1}^T \left[ \ln \det f(\omega_j, \theta) + \text{tr} \left( f^{-1}(\omega_j, \theta) I_T(\omega_j, y) \right) \right]$$

for equispaced frequencies  $\omega_j = \frac{2\pi j}{T}, j = 1, 2, \dots, T/2 - 1$  and the  $N \times N$  periodogram matrix  $I_T(\omega_j, y)$  defined as in (1). The  $N \times N$  matrix  $f(\omega_j, \theta)$  is the spectrum of the process to estimate. The spectrum of the VARFIMA process (2), (3) at frequency  $\omega_j$  is given by:

$$f(\omega_j, \theta) = (2\pi)^{-1} D(e^{i\omega_j})^{-1} \left( I_N - F(e^{i\omega_j}) \right)^{-1} \Omega \left( I_N - F(e^{-i\omega_j}) \right)^{-1} D(e^{-i\omega_j})^{-1}$$

where  $i$  is the imaginary unit and  $D(e^{-i\omega_j})$  is the complex conjugate of  $D(e^{i\omega_j})$ ;

$F(e^{i\omega_j}) = F_1 e^{i\omega_j} + F_2 e^{i2\omega_j} + \dots + F_p e^{ip\omega_j}$  and  $F(e^{-i\omega_j})$  is its complex conjugate.

Following standard practice, frequency  $\omega = 0$  is excluded from estimation in order to avoid singularity problems in the sample periodogram in the fractionally integrated framework.

### 3. Empirical Evidence

In this section we apply the framework discussed above for the estimation of a small quarterly model of the U.S. economy. Inflation is measured as the log-difference of the GDP chain-type price index, as in Stock and Watson (2007). For the VARFIMA we also condition on unemployment and nominal interest

rate<sup>7</sup>. Unemployment is measured by civilian unemployment rate. The original monthly series is converted to quarterly by sampling the middle month of each quarter. Nominal interest rate is measured by the secondary market rate three-month Treasury bill expressed as yield to maturity. Monthly data is also converted to quarterly by selecting the first month of each quarter, in order to align the interest rate data with inflation (see e.g. Cogley and Sargent (2002,2005)). All data span from 1948:1 to 2009:4 and were downloaded from the Federal Reserve Economic Database (FRED)<sup>8</sup>. For the sake of comparability with the existent time varying VAR literature, we work with a fixed lag-length VAR. To this respect we found that one lag in the autoregressive part plus the fractional integration component  $D(L)$ , provides the overall best fit<sup>9</sup> with the significant autoregressive component at each of the time dependent frequency domain estimations.

We identify the monetary policy shock by ordering interest rates down in the VAR. It is, we assume that policy shocks require at least one quarter percolating through non-policy variables. This short-run identifying assumption is standard in the VAR literature (see e.g. Leeper, Sims and Zha (1996), Christiano, Eichenbaum and Evans (1999) or Primiceri (2005)). We further assume that unemployment does not affect inflation contemporaneously as in Primiceri (2005) so the variables are arranged in the order  $y_t = (\pi_t, u_t, i_t)'$ . Different order inside the non-policy block does not alter results.

With the intention to shed some light on the contribution of the Fed to the different inflation episodes, we organize the results around four general

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<sup>7</sup> In this way our system is comparable to Cogley and Sargent (2002, 2005), Cogley et al. (2010), Primiceri (2005) or Benati and Surico (2006).

<sup>8</sup> <http://research.stlouisfed.org/fred2/>

<sup>9</sup> Note that the FI component delivers an infinite VAR representation.

themes: (i) what was the degree of inflation persistence and did it remain constant?; (ii) were the exogenous perturbations higher during the great inflation?; (iii) were differences in the transmission mechanism of policy shocks?; (iv) was the systematic part of monetary policy different during the great disinflation

### **3.1 Inflation persistence**

The persistence of inflation is a central concern of macroeconomics (see e.g. Fuhrer (2009) for a good review of the topic). Persistence is a key factor in the design of monetary policy since it determines the monetary policy transmission mechanism and also influences the behavior of private agents. Inflation persistence may also affect Fed beliefs about the existence of an exploitable trade-off between inflation and unemployment since tests of the natural rate hypothesis are very sensitive to it<sup>10</sup>. Recently, several authors have looked for evidence that changes in the behavior of central banks have resulted in a reduction of persistence. For instance, Cogley and Sargent (2002, 2005) using a multivariate time-varying VAR find that persistence increased in the early 1970's, remained high for around a decade and declined afterwards. On the other hand, Sims (2002) and Stock (2002) find that persistence remained constant and high over the past 40 years. This view is also supported by Pivetta and Reis (2007). From the other side, Benati and Surico (2008) and Cogley et al (2010) find a reduction in persistence in inflation gap<sup>11</sup>, although they agree that inflation has remained highly persistent. Already in the FI framework, Gadea and Mayoral (2006) test for changes in persistence using a Lagrange Multiplier test

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<sup>10</sup> Solow-Tobin type tests have been criticized to tend to reject the natural rate when inflation persistence is low (see e.g. Sargent 1971)

<sup>11</sup> They define the gap as the difference between inflation and the Federal Reserve's long-run target for inflation.

on the stability of the memory parameter of inflation. The authors find weak support to the hypothesis that persistence has changed.

The evolution of the memory parameter for inflation is provided in Figure 2. Several results stand out. First, as in Cogley and Sargent (2002, 2005), persistence increased during the 60's and 70's, reaching its top during early 80's, and have decrease during second half of the 80's and 90's, coinciding with the disinflation period. A new re-point of persistence seems to have appeared from the beginning of the millennium, but the statistical evidence about it is weak. Second, inflation has remained highly persistent during the whole period, with the long memory parameter always over 0.5. This means that inflation is characterized by a non-stationary but mean-reverting behavior; the effect of the shocks, although eventually disappear, decay at slow hyperbolic rate. This last result is consistent with the findings of Baillie et al. (1996) or Gadea and Mayoral (2006) in the FI framework, and provides evidence that inflation is not good characterized by an  $I(0)$  process.

### **3.2 The pattern of exogenous shocks**

Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) found evidence that U.S. economy from the early 80's experienced a growing period of stability, which they characterized in terms of a decline of the VAR innovation variances, phenomenon that is known as the "Great Moderation". It has been argued that neglecting the heteroskedasticity of the innovations may lead to an exaggerated parameter drift (see e.g. Sims (2002) or Stock (2002)). In the method proposed in this paper, the entire models parameters, including the parameters of the variance covariance matrix of the innovations are allowed to vary. Figure 3 presents the time profile for the standard deviation of the VAR

innovations together with their confidence intervals. The pattern resembles other estimates in the literature (see Bernanke and Mihov (1998), Cogley and Sargent (2005) or Primiceri). The standard deviations of the innovations of inflation and unemployment (Fig. 4.a and 4.b) have been falling strongly from the middle 80's, coinciding with the disinflation period. This result suggests that changes in the variances of exogenous shocks may be responsible of a considerable amount of the observed inflation stability during the 80's and 90's. Nevertheless, the standard deviation of the inflation innovation presents a less pronounced peak during the last 70's beginning of 80's compared to other works. As shown in the previous section, inflation almost acquired a unit root during the same period. Therefore, the strong peak found in previous literature may be in part explained by an exaggerated variance response due to the  $I(0)$  restriction.

### **3.3 Non-systematic monetary policy**

As in Primiceri (2005), the term non-systematic policy captures the responses of interest rate to policy mistakes and to variables other than inflation or unemployment. The identified monetary shock is therefore the logical measure of the non-systematic policy. Figure 3.c depicts the evolution of the standard deviation of the innovations to the interest rate. As can be seen in the figure, the standard deviation of the exogenous shocks has been also decreasing since the end of Volker's monetary aggregate targeting. Fixed policy rules, as the one stated in this paper<sup>12</sup>, are better approximations for monetary policy actions from the 80's on, which is consistent with the abandonment of discretionary macroeconomic policy by the Fed and the adoption of a rules-based macroeconomic policy.

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<sup>12</sup> See equation (8) in next section for an interpretation of the third equation of the VARFIMA as an augmented Taylor Rule.

The effects of monetary policy shocks on inflation and unemployment are summarized in Table 1. The table reports the IRF of the non-policy variables to the monetary shock for selected years. The selected years are 1975, 1984, 1996 and 2008. The dates coincide with the middle of the Burns-Miller's, Volker's, Greenspan's, and Bernanke's chairmanships period. Note also that 1984 coincides with the adoption of inflation-targeting regime by Volker and the end of the great inflation. As can be seen in the table, the responses of inflation and unemployment to a positive interest rate shock have the expected sign; negative for inflation and positive for unemployment for all selected years. Also, there are not significant differences in the size of the responses. In general all are small and statistically not different from zero. Nevertheless, there is some evidence of an increase of the long run responsiveness of unemployment to the policy shock in recent years. However, once one takes into account the decrease in the size of the shocks, as depicted in Figure 3.c, this increase become in line with the size of non-systematic policy actions.

#### **3.4 Systematic monetary policy.**

Along this section, we try to evaluate the degree of activism of the monetary policy, it is, how much the interest rate respond to inflation and unemployment movements. We find that the multivariate coherence provides a good measure of the systematic part of monetary policy. If  $X_t$  is a process with  $N > 2$  components, it may be the case that the most of the power in a given series can be removed by subtracting a linear function of several components. This would indicate a relationship among the components which might arise, for example, as a result of a common "driving mechanism".

Multiple coherence of interest rate  $i_t$  on inflation  $\pi_t$  and unemployment  $u_t$ , ranges between 0 and 1, and measures the portion of the power (density) at frequency  $\lambda$  attributable to the linear regression of  $i_t$  on  $m_t = \{\pi_t, u_t\}$ , and is given by :

$$\hat{R}_{j,m}^2(\lambda) = \hat{f}_{i,m}(\lambda) \hat{f}_m(\lambda)^{-1} \hat{f}_{i,m}(\lambda)^* / \hat{f}_i(\lambda),$$

Figure 4 depicts the (time varying) multiple coherence between the interest rate and the non-policy block. Results for selected years and frequencies, together with their confidence intervals may be found in the Table 2. As can be seen in the table, interest rate was explained by other two variables worse during the great inflation, especially at low and business cycle frequencies. More generally, the multiple coherence coefficient fall during Burns-Miller chairmanship, and rise strongly from the early 80's on. This pattern can be better seen in Figure 5, which depicts the time profile of the multiple coherence coefficients at business cycle frequencies.

Now we proceed to check if they have been changes in the size of the responses of interest rates to changes in the non-policy block during the two periods. To be concrete, consider the reduced form VARFIMA model given by (2) and (3). Substitution of the first equation into the second, and taking into account that  $A^{-1}\zeta_t = \varepsilon_t$ , leads to:

$$D(L)y_t = F(L)D(L)y_t + A^{-1}\zeta_t.$$

Multiplication of the both sides of the equation by  $A$  implies

$$AD(L)y_t = AF(L)D(L)y_t + \zeta_t.$$

Define  $AF(L) = G(L)$ . The equation for the interest rate (the third equation in the model) is given by:

$$\begin{aligned}
& a_{31}(1-L)^{d_1} y_{1,t} + a_{32}(1-L)^{d_2} y_{2,t} + (1-L)^{d_2} y_{3,t} = \\
& = G_{31}L(1-L)^{d_1} y_{1,t} + G_{32}L(1-L)^{d_2} y_{2,t} + G_{33}L(1-L)^{d_3} y_{3,t} + \zeta_{3,t}
\end{aligned}$$

Rearranging this expression leads to:

$$y_{3,t} = \frac{G_{31}L - a_{31}}{1 - G_{33}L} (1-L)^{d_1 - d_3} y_{1,t} + \frac{G_{32}L - a_{32}}{1 - G_{33}L} (1-L)^{d_2 - d_3} y_{2,t} + v_{3,t}$$

After collecting the polynomials, this expression can be re-written as:

$$y_{3,t} = \Phi_1(L) y_{1,t} + \Phi_2(L) y_{2,t} + v_{3,t} \quad (7)$$

where  $v_{3,t} = (1 - G_{33}L)^{-1} (1-L)^{-d_3} \zeta_{3,t}$ . The equation (8) can be considered as an augmented Taylor Rule. The coefficients of the polynomial  $\Phi_1(L)$  are the responses of the interest rate to the impulse in inflation. The sum  $\Phi_1(1)$  is finite if and only if  $|d_1 - d_3| < 1$ . In this framework, the Taylor principle<sup>13</sup> requires the infinite sum of these coefficients,  $\Phi_1(1)$  is greater than one. Figure 6 plots the evolution of the responses of interest rate to a 1% permanent increase in inflation after 1, 5 and 20 quarters. Some results must be highlighted. First, the response of interest rate to inflation is gradual. This gradual Fed response is also well documented in the literature (see e.g. Primiceri (2005)). Second, the systematic monetary policy becomes more reactive to inflation starting from middle 80's during Volker's chairmanship, and coinciding with the beginning of the great disinflation leading to the higher stability period. Figure 7 depicts the cumulative response up to 40 quarters, which proxies the long-run response, for selected years. As can be seen in the figure, we don't find significant differences in the

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<sup>13</sup> The Taylor principle specifies that for each one-percent increase in inflation; the central bank should raise the nominal interest rate by more than one percentage point. It has been argued that the Taylor Principle is a necessary and sufficient condition under which rational expectations equilibria exhibits desirable properties.

responses of years 1975, 1984, or 2006 but the response in the middle 90's, corresponding to the years of higher stability, is considerably stronger. In particular it reaches the Taylor principle before 20 quarters. Third, the Fed responses to inflation have declined strongly in recent years. Results for selected years together with their confidence intervals may be found in Table 3. Overall, we found strong evidence that the Fed monetary policy during second half of the 80's and 90's was significantly much more active towards inflation, even after controlling for heteroskedasticity in the disturbances. However, we do not find differences in the responses of interest rate to inflation under the Burns-Miller and Bernanke Chairmanships.

#### **4. Concluding Comments**

This paper applies frequency domain methods to study inflation persistence and changes in Fed's monetary policy without relying in the standard  $I(0)$  assumption over inflation (and the other variables in the VAR), consistent with the increasing evidence that inflation is much more persistent.

As in the previous literature, we find a strong reduction of the variance of the shocks which is in line with the view that a sizable amount of the great performance in the economy during the 80's and 90's is just a tale of good luck. However, even controlling for the (strong) reduction in the variance of the innovations, we find strong evidence of changes in the way the monetary policy has been conducted during the period considered, with an increasingly higher policy response to inflation starting from Volker's and extending up to middle Greenspan's chairmanships, as in Cogley and Sargent (2002, 2005) or Fernandez-Villaverde et al. (2010). This increasingly higher monetary response to inflation coincides with a reduction in the inflation persistence, which is often

interpreted as the consequence of a more active behaviour towards inflation from the part of the Central Bank. This result suggests that it still may be a role for the Fed in accounting for the different inflation episodes.

From the methodological point of view, the estimation of the time varying spectral density shows evidence of variation not only at low but also at the higher frequency band of the spectrum. Thus, it may well be the case that fixed lag length models, as the one employed in this paper and the others in the literature are not the best possible approximations to each period dynamics. Given that, contrary to the standard time varying VAR literature, the framework discussed here allows for pre-testing VAR length at each step, an appealing line of research would be to allow the autoregressive length at each period to change. Interesting different short-run dynamics of the inflation gap may appear from such exercise. We leave this question open for future research.

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## Appendix I: Tables and Figures

**Table 1 IRFs of variables to an interest rate shock after 1, 4, and 20 quarters, selected years**

|  | 1975   | 1984  | 1996  | 2008  |
|--|--|---|---|---|
| IRFs of inflation to an interest rate shock    |  |   |   |   |
| 1 quarter                                      | -0.0447<br>[-0.0841;0.0052]<br>[-0.0768;-0.0021] | -0.0044<br>[-0.0417;0.0242]<br>[-0.0363;0.0189] | -0.0076<br>[-0.0320;0.0137]<br>[-0.0276;0.0099] | -0.0159<br>[-0.0437;0.0136]<br>[-0.0390;0.0089] |
| 4 quarters                                     | -0.0141<br>[-0.0376;0.0153]<br>[-0.0333;0.0110]  | -0.0138<br>[-0.0365;0.0045]<br>[-0.0331;0.0012] | -0.0013<br>[-0.0126;0.0080]<br>[-0.0109;0.0063] | 0.0049<br>[-0.0146;0.0252]<br>[-0.0114;0.0220]  |
| 20 quarters                                    | -0.0086<br>[-0.0285;0.0164]<br>[-0.0248;0.0127]  | -0.0118<br>[-0.0324;0.0044]<br>[-0.0294;0.0014] | -0.0011<br>[-0.0096;0.0058]<br>[-0.0084;0.0046] | 0.0020<br>[-0.0097;0.0141]<br>[-0.0077;0.0122]  |
| IRFs of unemployment to an interest rate shock |  |   |   |   |
| 1 quarter                                      | 0.0348<br>[-0.0189;0.0896]<br>[-0.0101;0.0807]   | 0.0708<br>[0.0205;0.1128]<br>[0.0280;0.1053]    | 0.0270<br>[-0.0059;0.0544]<br>[-0.0010;0.0495]  | 0.0891<br>[0.0580;0.1127]<br>[0.0625;0.1082]    |
| 4 quarters                                     | 0.0638<br>[-0.0223;0.1531]<br>[-0.0080;0.1388]   | 0.1097<br>[0.0244;0.1883]<br>[0.0378;0.1749]    | 0.0384<br>[-0.0117;0.0813]<br>[-0.0041;0.0737]  | 0.1633<br>[0.0973;0.2206]<br>[0.1073;0.2106]    |
| 20 quarters                                    | 0.0452<br>[-0.0180;0.1116]<br>[-0.0074;0.1011]   | 0.0959<br>[0.0163;0.1700]<br>[0.0288;0.1575]    | 0.0678<br>[-0.0189;0.1234]<br>[-0.0073;0.1118]  | 0.2247<br>[0.1226;0.3132]<br>[0.1382;0.2976]    |

**Notes:** a) The selected years coincide with the middle period chair at Burns-Miller, Volker, Greenspan and Bernanke chairmanships. b) Numbers inside brackets are the 95 and 90% confidence intervals respectively.

**Table 2 Multiple coherence, 2 quarters, 3 quarters, 1 year, 5 years, 7 years frequency, selected years.**

|                      | 1975   | 1984   | 1996   | 2008   |
|----------------------|--|--|--|--|
| MULTIPLE COHERENCE   |  |  |  |  |
| 7 years frequency    | 0.4457<br>[0.2452;0.6864]<br>[0.2812;0.6504] | 0.3498<br>[0.1713;0.5861]<br>[0.2051;0.5523] | 0.4701<br>[0.3251;0.6198]<br>[0.3491;0.5958] | 0.6807<br>[0.5211;0.8284]<br>[0.5462;0.8033] |
| 5 years frequency    | 0.4072<br>[0.2291;0.6172]<br>[0.2607;0.5855] | 0.3336<br>[0.1734;0.5403]<br>[0.2034;0.5104] | 0.4645<br>[0.3226;0.6109]<br>[0.3462;0.5874] | 0.6616<br>[0.5053;0.8036]<br>[0.5296;0.7792] |
| 1 year frequency     | 0.0937<br>[0.0532;0.1501]<br>[0.0611;0.1422] | 0.1362<br>[0.0814;0.1889]<br>[0.0902;0.1801] | 0.2194<br>[0.1541;0.3069]<br>[0.1665;0.2944] | 0.1324<br>[0.0531;0.2143]<br>[0.0662;0.2011] |
| 3 quarters frequency | 0.0404<br>[0.0169;0.0897]<br>[0.0229;0.0838] | 0.0835<br>[0.0378;0.1367]<br>[0.0459;0.1286] | 0.1050<br>[0.0478;0.2022]<br>[0.0604;0.1896] | 0.0309<br>[0.0053;0.0771]<br>[0.0111;0.0712] |

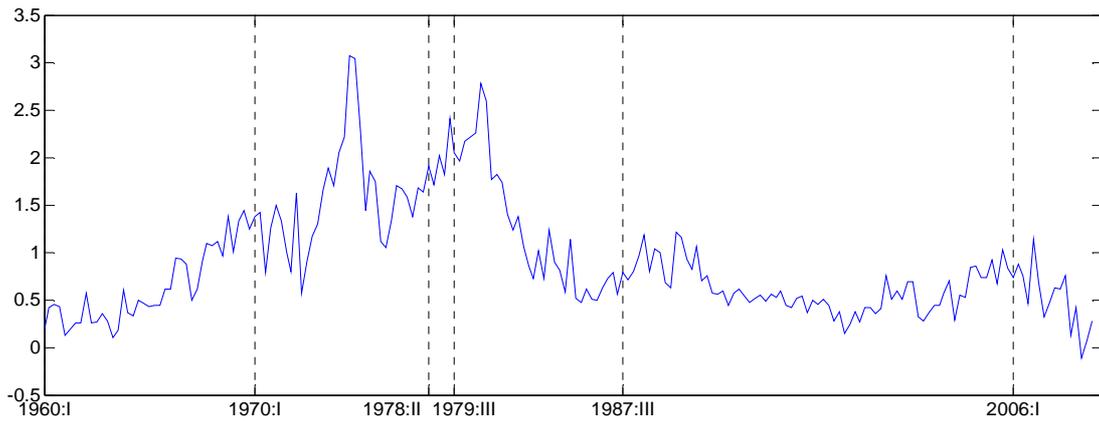
**Notes:** Numbers inside brackets are the 95 and 90% bootstrapped confidence intervals respectively.

**Table 3 IRFs (and their confidence intervals) of interest rate to a permanent change in inflation and unemployment after 1, 4, and 20 and 40 quarters for specific years**

|             | 1975  | 1984   | 1996   | 2008   |
|-------------|---|--|--|--|
| 0 quarter   | 0.1650<br>[0.0232;0.3165]<br>[0.0471;0.2925]  | 0.1682<br>[-0.0298;0.3177]<br>[-0.0014;0.2894] | 0.0734<br>[-0.0342;0.1766]<br>[-0.0170;0.1594] | 0.0066<br>[-0.0692;0.0820]<br>[-0.0568;0.0697] |
| 4 quarters  | 0.2359<br>[0.0210;0.4346]<br>[0.0547;0.4008]  | 0.2758<br>[-0.0203;0.4779]<br>[0.0204;0.4373]  | 0.3809<br>[0.1329;0.6222]<br>[0.1728;0.5823]   | 0.1170<br>[-0.0762;0.3192]<br>[-0.0439;0.2869] |
| 20 quarters | 0.2829<br>[-0.0046;0.5593]<br>[0.0414;0.5133] | 0.3240<br>[-0.0529;0.5920]<br>[-0.0003;0.5394] | 0.8394<br>[0.2313;1.4733]<br>[0.3327;1.3719]   | 0.2111<br>[-0.1431;0.5902]<br>[-0.0833;0.5303] |
| 40 quarters | 0.3060<br>[-0.0289;0.6372]<br>[0.0255;0.5829] | 0.3470<br>[-0.0756;0.6575]<br>[-0.0158;0.5977] | 1.1745<br>[0.2453;2.1775]<br>[0.4030;2.0197]   | 0.2670<br>[-0.1985;0.7715]<br>[-0.1193;0.6923] |

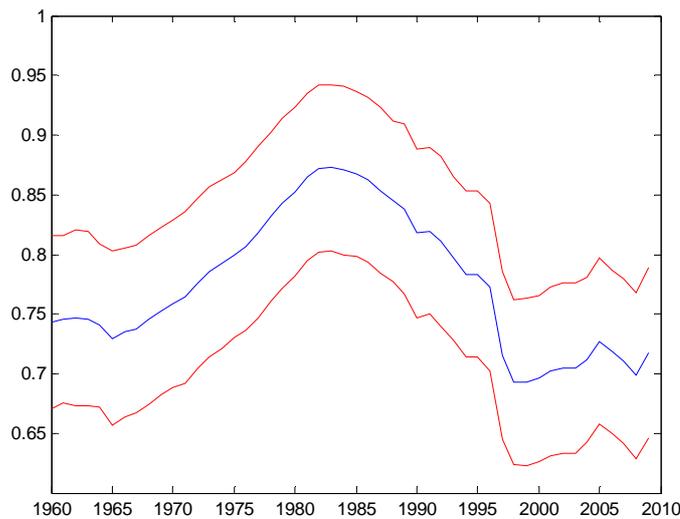
Notes: Numbers inside brackets are the 95 and 90% bootstrapped confidence intervals respectively.

**Fig.1 U.S Quarterly GDP Inflation**



**Notes:** a) Inflation is measured as the log-difference of a quarterly GDP chain price index. B) Fed chairmanships: McChesney (1951:II-1970:I), Burns (1970:I-1978:II), Miller (1978:II-1979:III), Volker (1979:III-1987:III), Greenspan (1987:III-2006:I), Bernanke (2006:I-pres.).

**Fig.2 Time profile of the estimated long memory parameter**



**Notes:** Time profile of the fractional integration parameter at zero frequency-BLUE:  $d$ . Stationarity requires  $0 \leq d < 0.5$ , strictly greater than 0 for long memory. Non-stationary but mean reversion behavior appears when  $0.5 \leq d < 1$ , implying high persistence but with the effect of the shocks dying in the long run although not fast enough to deliver finite variance. Permanent effect of the shocks appears whenever  $d \geq 1$ . 16% and 84% bootstrapped percentiles-RED

### Fig.3 Time profiles of the standard deviations of the residuals

Fig.4.a Std. inflation shock

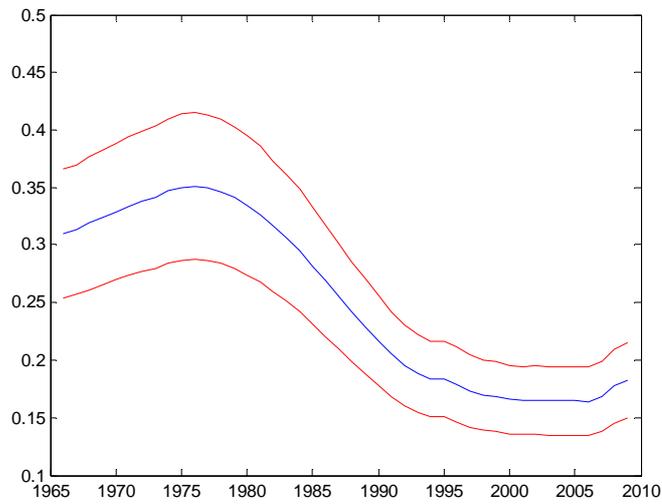


Fig.4.b Std. unemployment shock

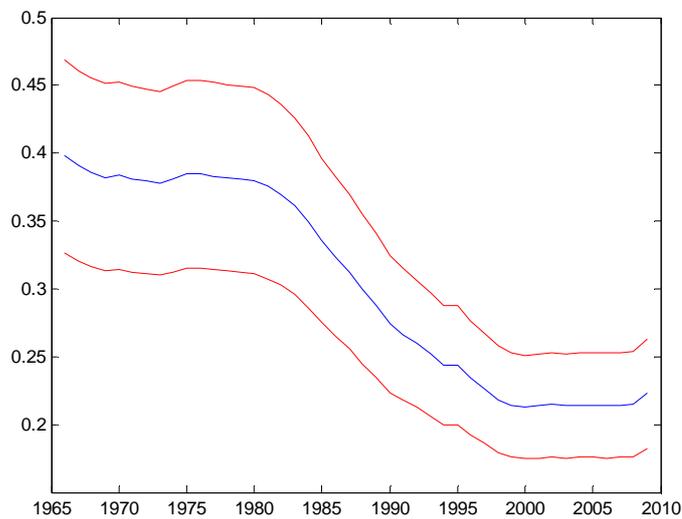
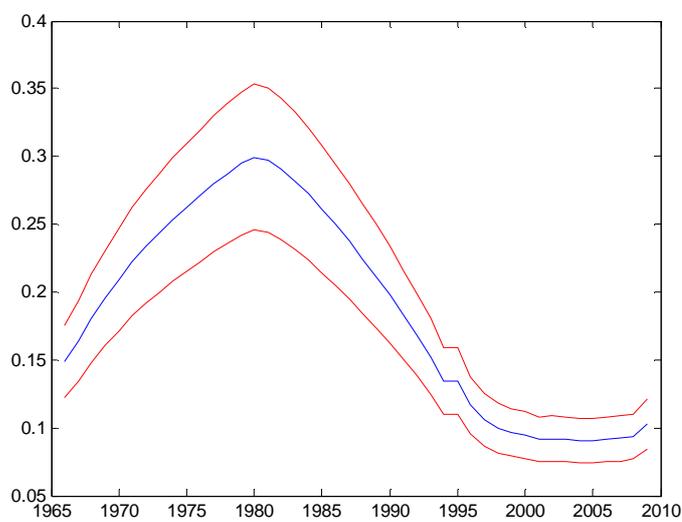
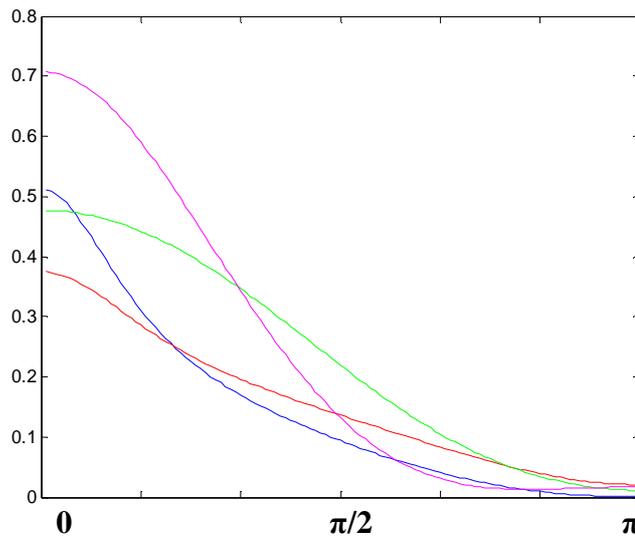


Fig 4.c Std of interest rate shock



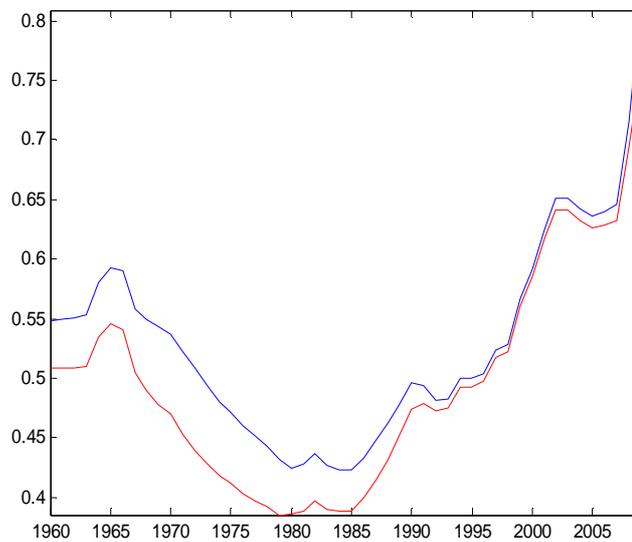
**Notes:** Standard deviations of VARFIMA innovations-BLUE, 95% bootstrapped percentiles-RED

**Fig.4 Multiple coherence at different frequencies for selected years, VARFIMA**



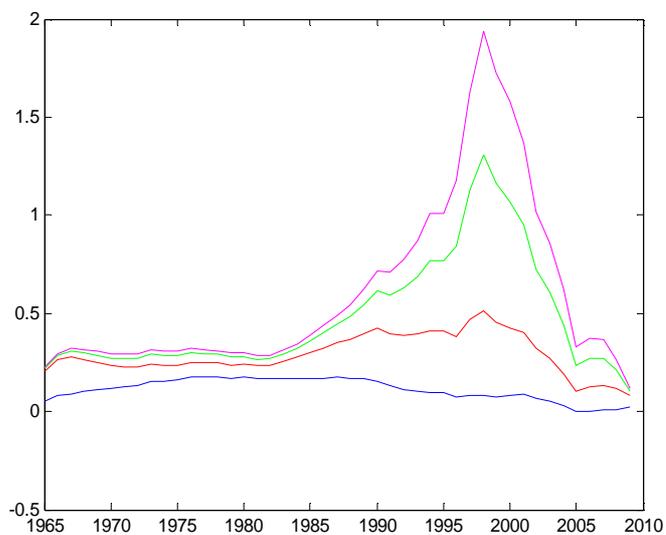
**Notes:** a) years: 1975-BLUE, 1984-RED, 1996-GREEN, 2008-PURPLE; b) Multiple coherence estimated from the VARFIMA spectrum.

**Fig.5 Time profile, Multiple coherence at 5 and 7 year frequencies, VARFIMA**



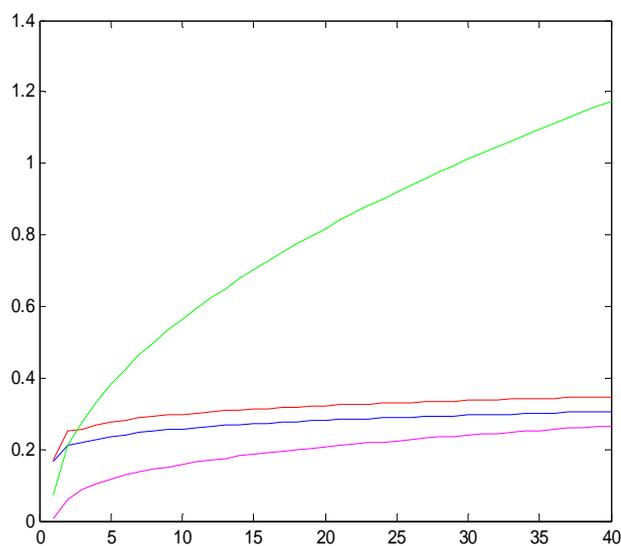
**Notes:** a) Frequencies correspondent to: 7 years-BLUE, 5 years-RED; b) Multiple coherence estimated from the VARFIMA spectrum.

**Fig.6 Time profile of the responses of interest rate to a 1% permanent increase in inflation, Contemporaneous response, 1 year, 5 year and 10 years, VARFISMA**



**Notes:** Contemporaneous response-BLUE, 1 year response-RED, 5 years response-GREEN, 10 years response-PURPLE.

**Fig.7 Responses of interest rate to a 1% permanent increase in inflation for selected years, VARFISMA**



**Notes:** Years: 1975-BLUE, 1984-RED, 1996-GREEN, 2008-PURPLE