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**Número d'expedient**

2007 BE-100003

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**Paraules clau:** cal que esmenteu cinc conceptes que defineixin el contingut de la vostra memòria.  
fluctuacions del buit quàntic, efecte Casimir, teoria de Fulling-Davies, energia fosca, constant cosmològica

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**Data de presentació de la justificació**

07-02-2008

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Nom i cognoms i signatura  
del/de la investigador/a

Vistiplau del/de la responsable de la  
sol·licitud





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**Resum del projecte:** cal adjuntar dos resums del document, l'un en anglès i l'altre en la llengua del document, on s'esmenti la durada de l'acció

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**Resum en la llengua del projecte** (màxim 300 paraules)

L'acció s'ha dut a terme al Departament de Física i Astronomia de la Facultat d'Arts i Ciències de Dartmouth College, NH, USA. Ha estat ben fructífera, des de diversos punts de vista: científic, filosòfic i humà. Hem avançat, durant els sis mesos, en la comprensió del comportament de les fluctuacions del buit quàntic en presència de fronteres, fixes i mòbils, i en situacions en que canvia la topologia de l'espai-temps. En particular, hem refet la deducció de l'efecte Casimir dinàmic, corregint defectes en l'adequada renormalització (apareguts en tot un seguit de deduccions prèvies). I hem anat més enllà, en el càlcul de la producció de partícules en tal efecte, veient que l'estadística pot fluctuar d'acord amb les condicions imposades, resultat molt remarcable. Hem aprofundit en els models alternatius a la gravetat d'Einstein, en que la curvatura  $R$  ve substituïda per una funció  $f(R)$  considerant, en particular, models viables de gravetat, tant en l'esquema de Jordan com en el d'Einstein (que si bé són equivalents matemàticament no ho són pel que fa a la física a la que donen lloc). Hem considerat una classe de gravitats modificades realístiques de tipus exponencial, havent fet especial atenció als models de tipus esglaó prèviament considerats a la literatura i que nosaltres hem tractat de manera completa. Aquests resultats són molt prometedors des del punt de vista fenomenològic i a efectes de classificació de tots els models de gravitació admissibles. Hem considerat explícitament models d'un i de dos esglaons, però l'esquema desenvolupat permet la seva extensió a un cas general d' $N$  esglaons. Els models unifiquen sota un mateix esquema qüestions crucials d'inflació primordial i de l'expansió accelerada actual. I són capaços de passar fàcilment tots els tests, gens trivials, que han exclòs diversos models prèviament. Tota aquesta activitat ens obre un futur prometedori i serà continuada en treballs successius, el que justifica ja una nova estada de recerca.

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**Resum en anglès** (màxim 300 paraules)

The action has been carried out at the Department of Physics and Astronomy, Faculty of Arts and Sciences of Dartmouth College, Hanover, NH, USA. It has been very successful, from different viewpoints: scientific, philosophical, human. We have definitely advanced, during the past six months, towards the comprehension of the behavior of the fluctuations of the quantum vacuum in the presence of boundaries, moving and non-moving, and also in situations where the topology of space-time changes: the dynamical Casimir effect, regularization problems, particle creation statistics, according to different BC, etc. We have solved some longstanding problems and got in this subject quite remarkable results (as we will explain in more detail below). We also pursued a general approach towards a viable modified  $f(R)$  gravity in both the Jordan and the Einstein frames (which are known to be mathematically equivalent, but physically not so). A class of exponential, realistic modified gravities has been introduced by us and investigated with care. Special focus was made on step-class models, most promising from the phenomenological viewpoint and which provide a natural way to classify all viable modified gravities. One- and two-steps models were considered, but the analysis is extensible to  $N$ -step models. Both inflation in the early universe and the onset of recent accelerated expansion arise in these models in a natural, unified way, what makes them very promising.

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**Resum en anglès** (màxim 300 paraules) – continuació -.

Moreover, it is demonstrated in our work that models in this category easily pass all local tests, including stability of spherical body solution, non-violation of Newton's law, and generation of a very heavy positive mass for the additional scalar degree of freedom. All this opens very nice possibilities for future research, which already justify a new action and research stay at Dartmouth College.

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**2.- Memòria del treball** (informe científic sense limitació de paraules). Pot incloure altres fitxers de qualsevol mena, no més grans de 10 MB cadascun d'ells.

The work carried out during the action can be classified into different subjects, in the end into two main ones, that are in fact quite correlated in the background. On the one hand, the dynamical Casimir effect, regularization problems, particle creation statistics, according to different BC, etc. On the other, a general approach towards a viable modified  $f(R)$  gravity in both the Jordan and the Einstein frames (which are known to be mathematically equivalent, but physically not so). Both inflation in the early universe and the onset of recent accelerated expansion arise in these models in a natural, unified way, what renders them very promising.

**Dynamical Casimir Effect and a Black Hole collapse simulated by vacuum fluctuations**

Creation of scalar massless particles in two-dimensional Minkowski space-time—as predicted by the dynamical Casimir effect—has been studied in this program [1,3], for the case of a semitransparent mirror initially at rest, then accelerating for some finite time, along a trajectory that simulates a black hole collapse (defined by Walker, and Carlitz and Willey), and finally moving with constant velocity. When the reflection and transmission coefficients are those in the model proposed by Barton, Caloggeracos, and Nicolaevici [ $r(w)=-i a/(w+i a)$  and  $s(w)=w/(w+i a)$ , with  $a \geq 0$ ], the Bogoliubov coefficients on the back side of the mirror can be computed exactly. This allowed us to prove that, when  $a$  is very large (case of an ideal, perfectly reflecting mirror) a thermal emission of scalar massless particles obeying Bose-Einstein statistics is radiated from the mirror (a black body radiation), in accordance with results previously obtained in the literature. However, when  $a$  is finite (semitransparent mirror, a physically realistic situation) the striking result is obtained that the thermal emission of scalar massless particles obeys Fermi-Dirac statistics. We have also shown that the reverse change of statistics takes place in a bidimensional fermionic model for massless particles, namely that the Fermi-Dirac statistics for the completely reflecting situation will turn into the Bose-Einstein statistics for a partially reflecting, physical mirror. Let me now elaborate more on the details.

The Davies-Fulling model describes the creation of scalar massless particles by a moving perfect mirror following a prescribed trajectory. This phenomenon is also termed as the dynamical Casimir effect. Recently, the authors of the present paper introduced a Hamiltonian formulation in order to address some problems associated with the physical description of this effect in the time interval while the mirror is moving; in particular, of the regularization procedure, which turns out to be decisive for the correct derivation of physically meaningful quantities. A basic difference with previous results was that the motion force derived within the new approach contains a reactive term—proportional to the mirror's acceleration. This term is of the essence in order to obtain particles with a positive energy all the time while the oscillation of the mirror takes place, and which always satisfy the energy conservation law. Those result followed essentially from the introduction of

physically realistic conditions, e.g. a semi-transparent or partially transmitting mirror, which is perfectly reflecting for low frequencies but becomes transparent to very high ones.

Here [1] we studied a different aspect of the introduction of physically plausible, semitransparent mirrors, namely the particle spectrum produced—in the conditions of the Fulling-Davies effect—by a mirror of this sort which is initially at rest, then accelerates during a large enough (but finite) time span,  $u_0$ , along a trajectory that simulates a black hole collapse, as defined by Walker, and Carlitz and Willey, and finally, for  $u \geq u_0$ , is left alone moving with constant velocity in an inertial trajectory.

We were interested in calculating the radiation emitted by the mirror from its back (e.g. right) side. As is well-known, a perfect mirror that follows this kind of trajectory produces a thermal emission of scalar massless particles obeying Bose-Einstein statistics. Turning to the case of a partially reflecting mirror—in which we were mainly interested in our work—in order to obtain the radiation on its right hand side (rhs), we also need to calculate the Bogoliubov coefficient. We thus first obtain the 'in' modes on the rhs of the mirror for corresponding reflection and transmission coefficients. We have studied in detail the creation of scalar massless particles in a two-dimensional Minkowski space-time (the Davies-Fulling theory) and specifically for the case of a semi-transparent mirror, which reflects low frequency modes but is transparent to high enough frequencies, being the reflection and transmission coefficients analytic functions of the frequency. The considered mirror is initially at rest, then accelerates, during some finite time, along a trajectory that simulates a black hole collapse (as defined by Walker, and Carlitz and Willey), and finally rests moving with constant velocity. When the reflection and transmission coefficients are those in the model proposed by Barton, Calogheracos, and Nicolaevici, namely  $r(w) = -i a/(w+i a)$  and  $s(w) = w/(w+i a)$ , with  $a \geq 0$ , the Bogoliubov coefficients on the back side of the mirror could be computed exactly. This has allowed us to rigorously prove that, when  $a$  is very large (the case of an ideal, perfectly reflecting mirror) a thermal emission of scalar massless particles obeying Bose-Einstein statistics is radiated from the mirror (a black body radiation), in accordance with previous results in the literature. Moreover, we have also seen that when  $a$  is finite (the case of a semi-transparent mirror, that is, a physically realistic situation) the surprising result is obtained that the thermal emission of scalar massless particles obeys Fermi-Dirac statistics. We have also shown in detail that the reverse change of statistics takes place in a bidimensional fermionic model for massless particles, namely, that the Fermi-Dirac statistics for the completely reflecting situation gives rise to the Bose-Einstein statistics for the case of a semi-transparent, physical mirror [3].

The results we obtained [1,3] are absolutely solid—they do not hang on a perturbative expansion or approximation of any sort. The physical reason for this surprising change of statistics may be found in the fact that the form of the spectrum is actually determined not through the statistics of the field but rather by the specific trajectory of the mirror and by its interaction with the radiation field. The same kind of phenomenon occurs in the case of an electric charge following the trajectory  $v = (1/k)(1 - e^{-ku})$ . When the radiation field has spin 1, the radiation emitted by the charge obeys Bose-Einstein statistics, but when a scalar charge, and consequently an scalar radiation field, is considered, the emitted radiation will obey Fermi-Dirac statistics. Another situation where this kind of features occurs, is when one measures the spectrum of an scalar field using a DeWitt detector which follows a uniformly accelerated world-line in Minkowski space-time. In this case when the dimension of the space-time is even the Bose-Einstein statistics is obtained. However, when the dimension is odd, precisely the reverse change of statistics occurs in the emitted radiation. This research owes a lot to Prof. Roberto Onofrio's insights and discussions.

### De Sitter cosmology from Gauss-Bonnet dark energy with quantum effects

This is a related research which uses the same kind of quantum effects at cosmological scale. In our work, a Gauss-Bonnet dark energy model was considered, which is inspired in string or M-theory and takes also into account quantum contributions. Those are introduced from a conformal quantum anomaly. The corresponding solutions for the Hubble rate,  $H$ , were studied starting from the Friedmann-Robertson-Walker equation. It was seen that, as a pure effect of the quantum contributions, a new solution for  $H$  exists in some region, which does not appear in the classical case. The behavior of all encountered solutions is studied with care, in particular, the role played by the quantum correction term—which depends on the number of matter fields—on the stability of the solutions around its asymptotic value. It is argued in our work that, contrary to what happens in the classical case, quantum effects remarkably lead to the realization of a de Sitter stage which corresponds to the inflation and dark energy stages, even for positive values of the  $f_0$  constant (coupling of the field with the Gauss-Bonnet invariant).

To summarize the research done [5], a number of quite interesting conclusions could be drawn from our study of the influence of a combination of quantum effects and modified gravity as a possible way for interpreting dark energy in an accelerated inflationary de Sitter universe. In particular, it is expected that for large values of the curvature,  $R = 6 H' + 12 H^2$ , the de Sitter epoch thus described corresponds to early-time inflation. From another point of view, for small values of curvature,  $R \sim (10^{-33} \text{ eV})^2$ , we expect that the de Sitter era realized in this model will correspond to the dark energy stage. However, as was seen, this occurs for both positive and negative values of the potential, depending on the sign of  $f_0$ . This result clearly shows that, contrary to what happens in the classical case, quantum effects lead in fact to the realization of the de Sitter era corresponding to the dark energy stage, even for positive values of the  $f_0$  constant. Such situation may potentially have interesting cosmological consequences. In a similar fashion, the coupling of GB dark energy

with  $F(R)$  gravity could have been considered, too. This should not represent a big problem, in principle, owing to the fact that the curvature is constant on the solutions. That possibility deserves further investigation.

### Black hole entropy in modified gravity models

Here we carried out an analysis of some modified gravity models, based on the study of pure Schwarzschild and of Schwarzschild-de Sitter black holes, and involving the use of the Noether charge method. Corrections to the classical Einsteinian black hole entropy were seen to appear [4]. We showed explicitly how the condition of positive entropy can be used in order to constrain the viability of modified gravity theories. Indeed increasing interest is attracted by modified versions of general relativity. They have been proposed as very serious alternatives to Einstein's theory of gravitation, and can be used to describe more accurately the observed accelerated expansion of our universe. In addition, it has been shown that it is actually possible to reconstruct the explicit form of the postulated curvature function  $f(R)$ , from the universe expansion history. Modified versions of general relativity are known to be mathematically equivalent to scalar fields models, this meaning that a solution in a modified gravity model can always be mapped into a solution of the corresponding scalar field theory. In spite of this mathematical correspondence, physical equivalence does not always follow. In fact, two corresponding solutions of two equivalent theories can actually exhibit rather different physical behaviors. Furthermore, it is not necessary, in order to justify modified gravity, to do it by always using this relation with scalar field theories. Because of the new situation, in the following we will disregard this mathematical equivalence and do consider in our analysis modified gravity as an independent theory aiming directly at some measurable physical properties. What is more, in our treatment modified gravity was in fact viewed just as a different classical theory of gravitation. Although other models have been considered with the Gauss-Bonnet scalar in the action, we restricted our attention to pure  $f(R)$  models. As often discussed, there are limitations on the function  $f(R)$  when trying to construct a theory which is in agreement with the very precise solar system tests carried out so far, as well as with all the known cosmological bounds. Recently, different models of that kind have been studied, several of them having been reported to pass all solar system tests (as those of Sawicky and Hu, Starobinski, etc.); in addition, they exhibit a number of very interesting features. Possible Newton law corrections to such models have also been considered.

We carried out an analysis of these models based on the study of pure Schwarzschild and also Schwarzschild-de Sitter black holes (SBH, SdSBH), calculated through use of the Noether charge method. We started with a discussion of two examples and went over to study more recent ones. The direct confrontation of basic quantities, as the black hole entropy, with the well established, classical (Einsteinian) result offered us a further insight into the construction of a general  $f(R)$  theory. We started from modified  $f(R)$  gravity and the Noether charge method to compute the BH entropy and then calculated the BH entropy for the models mentioned. After that, we extended our analysis to other models in the recent literature, taking due care of the sign of the BH entropy and also discussing the stability conditions as well as the existence of a Schwarzschild BH solution. We concluded our work by providing a brief comparison of the different models considered [4].

Having analyzed the different suitable models, recently considered in the literature, and having shown explicitly how corrections to the 'classical' BH entropy can in fact appear, we also argued that indeed the condition of positive entropy can be used as an extra condition in order to constrain the viability of modified gravity theories. Of course this condition is equivalent to the requirement that neither ghost nor tachyon fields appear in the equivalent scalar field models. Anyhow, if referred to the BH entropy, this condition was seen by us [4] to have a direct interpretation in the framework of modified gravity, without needing to pass through the (mathematically but not physically equivalent) scalar field theories. This quite simple considerations may be very useful for future analysis.

### Modified $f(R)$ gravities describing inflation and accelerated expansion

A general approach to viable modified  $f(R)$  gravity was developed in this work [6] in both the Jordan and the Einstein frames. A class of exponential, realistic modified gravities was introduced and investigated with care. Special focus was made on step-class models, most promising from the phenomenological viewpoint and which provide a natural way to classify all viable modified gravities. One- and two-steps models were explicitly considered, but the analysis is extensible to  $N$ -step models. Both inflation in the early universe and the onset of recent accelerated expansion arise in these models in a natural, unified way. Moreover, it was demonstrated that models in this category easily pass all local tests, including stability of spherical body solution, non-violation of Newton's law, and generation of a very heavy positive mass for the additional scalar degree of freedom. In our investigation, a general approach to viable modified gravity has been developed in both the Jordan and the Einstein frames. We have focussed on the so-called step-class models mainly, since they seem to be most promising from the phenomenological viewpoint and, at the same time, they provide a natural possibility to classify all viable modified gravities. We have explicitly presented the cases of one- and two-step models, but a similar analysis can be extended to the case of an  $N$ -step model, with  $N$  being finite or countably infinite. No additional problems are expected to appear and the models can be adjusted, provided one can always find smooth solutions interpolating between the de Sitter solutions (what seems at this point a reasonable possibility), to repeat at each stage the same kind of de Sitter transition. We

can thus obtain multi-step models which may lead to multiple inflation and multiple acceleration, in a way clearly reminiscent of braneworld inflation.

The work done in this subject looks quite promising, with the added bonus that the model's construction is rather simple, as we have here shown explicitly. All the time, as a guide for an accurate analysis, use has been made of the simple but efficient tools provided by the corresponding toy model constructed with sharp distributions, a new technique that we have here introduced too. It is to be remarked that, for the infinite-step models, one can naturally expect to construct the classical gravity analog of the stringy landscape realizations, as in the classical ideal fluid model. For one of the models explicitly considered, both inflation in the early universe and the recent accelerated expansion could be understood in a unified way. If we start with large curvature,  $f(R)$  becomes almost constant and plays the role of the effective cosmological constant, which would generate inflation. For a successful exit from the inflationary epoch we may need, in the end, more (say small non-local or small  $R^n$ ) terms. When curvature becomes smaller, matter could dominate, what would indeed lower the curvature values. Then, when the curvature  $R$  becomes small enough and in a range of values of  $R$ ,  $f(R)$  becomes again an almost constant function, and plays the role of the small cosmological constant which generates the accelerated expansion of the universe, that started in the recent past. Moreover, the model naturally passes all local tests and can be considered as a true viable alternative to General Relativity.

Some remark is however in order in this sense. On general grounds, one is dealing here with a highly non-linear system and one should investigate all possible critical points thereof (including other time-dependent cosmologies), within the dynamical approach method. Of course, the existence of other critical points is possible; anyhow, for viable  $f(R)$  models, to find them is not a simple task, and we have here restricted our effort to the investigation of the  $dS$  critical points. With regard to the stability of these points, the one associated with inflation should be unstable. In this way, the exit from inflation could be achieved in a quite natural way. In particular, for instance, this is in fact the case for the two step model with the  $R^3$  term. In conclusion for this subject, we introduced a class of exponential, realistic modified gravities and investigated them in depth. Some of these models ultimately lead to the unification of the inflationary epoch with the late-time accelerating epoch, under quite simple and rather natural conditions. What remains to be done is to study those models in further quantitative detail, by comparing their predictions with the accurate astrophysical data coming from ongoing and proposed sky observations. It is expected that this can be done rather soon, having in mind the possibility to slightly modify the early universe features of the theories here introduced, while still preserving all of their nice, realistic current universe properties, as we have shown above. This research owes a lot to Prof. Robert Caldwell's insights and discussions.

## References

All these papers owe their existence to the program, and so is it recognized in the acknowledgments of each of them, where the financial help from AGAUR and the corresponding reference are quoted. Further references and detail are to be found in the reference sections of these papers.

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